L

S

$$|f\rangle = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad |g\rangle = \begin{pmatrix} z \\ w \end{pmatrix}$$

$$|f\rangle\langle g| = \begin{pmatrix} az^* & aw^* \\ bz^* & bw^* \\ cz^* & cw^* \end{pmatrix}, \quad |h\rangle = \begin{pmatrix} d \\ e \end{pmatrix}$$

$$|f\rangle\langle g|h\rangle$$

$$\begin{pmatrix} az^* & aw^* \\ bz^* & bw^* \\ cz^* & cw^* \end{pmatrix}\begin{pmatrix} d \\ e \end{pmatrix} = \begin{pmatrix} az^*d + aw^*e \\ bz^*d + bw^*e \\ cz^*d + cw^*e \end{pmatrix}$$

$$P(a|b) = |\langle a|b\rangle|^2$$
$$= |\langle a|U^\dagger U|b\rangle|^2$$
$$= |\langle a'|b'\rangle|^2$$
$$= P(a'|b')$$

$$y = Ax \qquad x = A^{-1}y$$

$$VA\underbrace{V^{-1}V}AV^{-1} \dots VAV^{-1}$$
$$VA^2V^{-1} \dots VA^2V^{-1}$$
$$VA^4V^{-1}$$

$$(^{-1} \;\; ()\,)$$

$$\eta = \begin{pmatrix} 0 & \ddots & \\ & & 1 \\ & & & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} & \\ & \end{pmatrix}, \quad A = \begin{pmatrix} & & \end{pmatrix}$$

$$\langle n|A|m\rangle = \sum_j \langle n|b_j\rangle s_j \langle a_j|m\rangle$$
$$= \sum_j U_{nj} s_j V^{+}_{mj}$$

$$A = \sum |b_j\rangle s_j \langle a_j|$$
$$= \sum_j \frac{A|a_j\rangle}{s_j} s_j \langle a_j| = \sum_j A|a_j\rangle\langle a_j|$$
$$= A \sum_j |a_j\rangle\langle a_j| = A I = A$$

$$A = \sum |a_j\rangle s_j \langle b_j|$$
$$= \sum |b_j\rangle s_j^* \langle a_j|$$

$S_j = S_j \quad |a_j\rangle = |\omega_j\rangle$

$$A = \sum |a_j\rangle S_j \langle a_j|$$

$$\Sigma = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\Sigma^+ = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$|\vec{r}\rangle = |x\rangle|y\rangle|z\rangle$$
$$= |r\rangle|\theta\rangle|\phi\rangle$$

$|+,+\rangle, \quad |+,-\rangle, \quad |-,+\rangle, \quad |-,-\rangle$

$|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$|gm\rangle = \sum_{ik} S_{ik} |i, k\rangle$$

$$|\ \rangle|\ \rangle = \sum_{ik} \phi_i \psi_k |i, k\rangle$$

$S_{ik} \quad n^2 \quad$ #'s

$\phi_i \; \psi_k \quad 2n$ #'s

$(+)$

$$E_i^{(+)}(x)\,|\{\alpha\}\rangle$$

$$= \sum_{k,s} e^{ikx}\, a(k,s)\, e_i(k,s)\,|\{\alpha\}\rangle$$

$$a(k,s)\,|\underline{\{\alpha\}}\rangle = \alpha(k,s)\,|\{\alpha\}\rangle$$

$$\{\alpha\} = \{\alpha(k_1,1), \alpha(k_1,2), \alpha(k_2,1), \alpha(k_2,2), \ldots\}$$

$$\sum_{k,s} e^{ikx}\alpha(k,s)\, e_i(k,s)\,|\{\alpha\}\rangle$$

annihilation

$$= \mathcal{E}_i^{+}(x)$$

$$A = |k\rangle\langle k|A|l\rangle\langle l|$$

$$= |k\rangle\langle k|b_j\rangle S_j\langle a_j|l\rangle\langle l|$$

$$\langle k|b_j\rangle S_j\langle a_j|l\rangle$$

$$A_{kj} = U_{kj}\,\Sigma_j\,V^{+}_{lj}$$

$$|b_j\rangle = A|a_j\rangle = \alpha_j|a_j\rangle$$

$$A|a_j\rangle = \alpha_j|a_j\rangle$$

$$\langle a_j|A|a_j\rangle = \alpha_j$$

---

if $A = A^\dagger$

then $A^\dagger A |a_j\rangle = S_j |a_j\rangle$

becomes $A^2 |a_j\rangle = S_j |a_j\rangle$

$A|a_j\rangle = \alpha_j |a_j\rangle$

$S_j = \alpha_j^2$

$A = USU^\dagger = U\Sigma U^\dagger$

$A^\dagger A = I$

$|k\rangle\langle k|$

$A|k\rangle = |b_k\rangle$

$A = \sum_k |b_k\rangle\langle b_k|A|k\rangle\langle k|$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

$$f = z^2 z^*$$

$$\frac{\partial f}{\partial z} = 2 z z^*$$

$$\frac{\partial f}{\partial z^*} = z^2$$

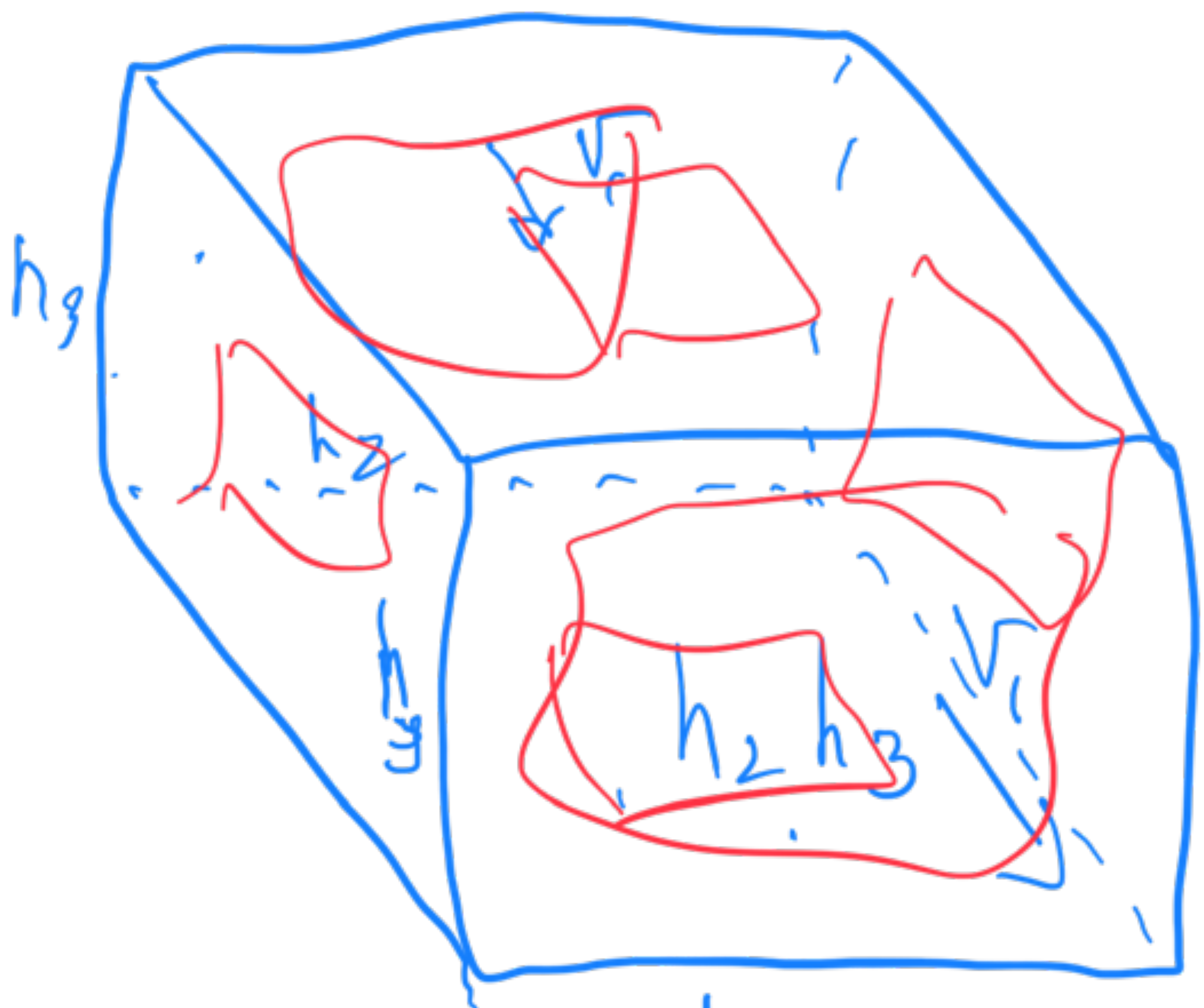
$$\frac{\partial f}{\partial x} = f_x = \frac{\partial f}{\partial z}\frac{\partial z}{\partial x} + \frac{\partial f}{\partial z^*}\frac{\partial z^*}{\partial x}$$

$$= \frac{\partial f}{\partial z} + \frac{\partial f}{\partial z^*}$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial z}\frac{\partial z}{\partial y} + \frac{\partial f}{\partial z^*}\frac{\partial z^*}{\partial y}$$

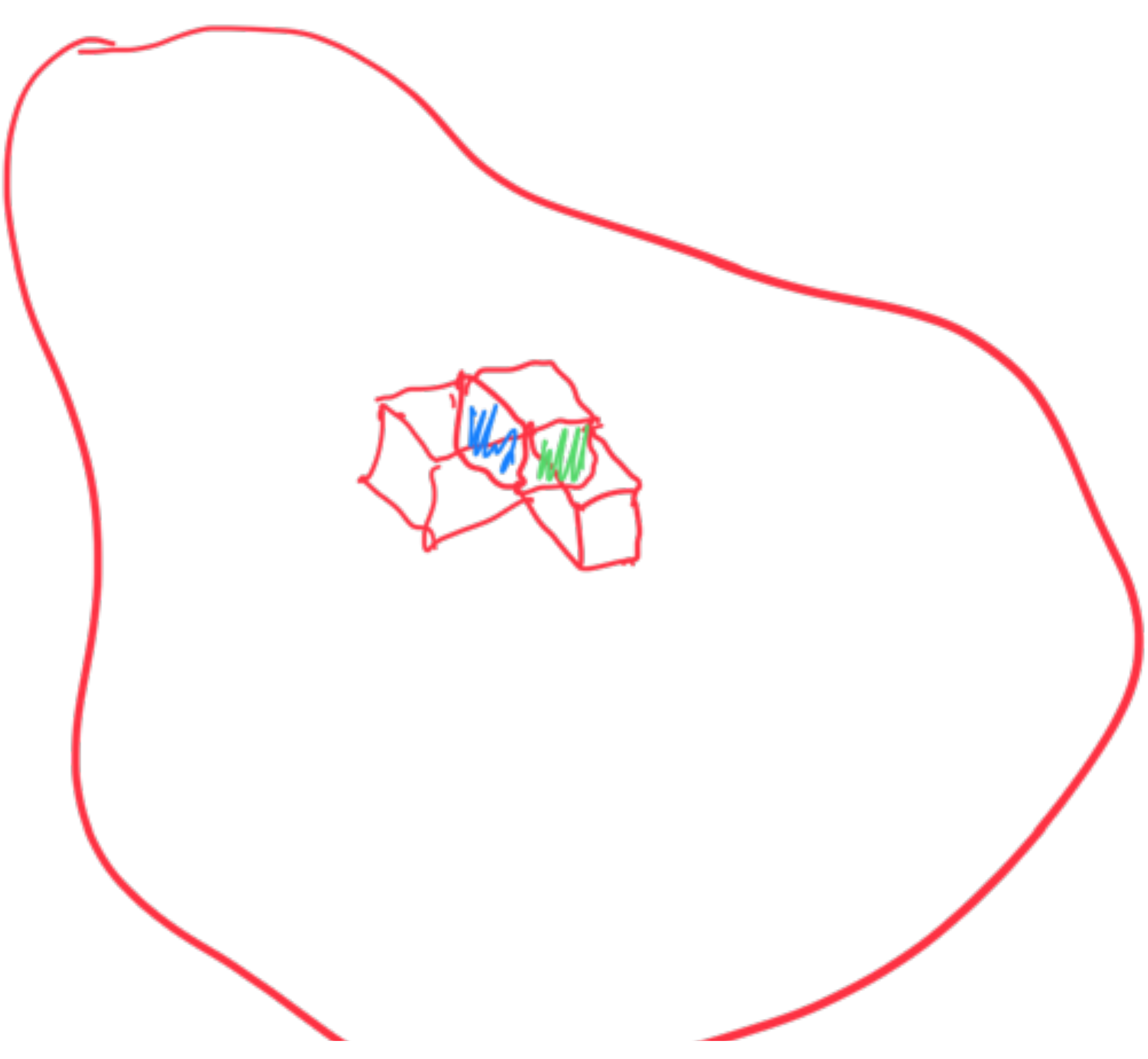
$$= i\,\partial f - i\,\partial f$$

$\partial z \qquad \partial z^*$



h_2

$$du_2 du_3 \frac{\partial (v_1 h_2 h_3)}{\partial u_1}$$



$\delta(\vec{r})$, $\delta^3(\vec{r})$

$\delta^{(3)}(\vec{r}) = \underline{\delta^{(3)}(\vec{r})}$

$a \times b$

$$\frac{\partial(h_j r_j)\, du_j\, du_1}{\partial u_i}$$

$B = \frac{1}{r^2 \sin\theta} \{$

$A = y\hat{x}$ $\qquad A_1 = y$

$\nabla \times A = \epsilon_{ijk}\partial_j A_k$

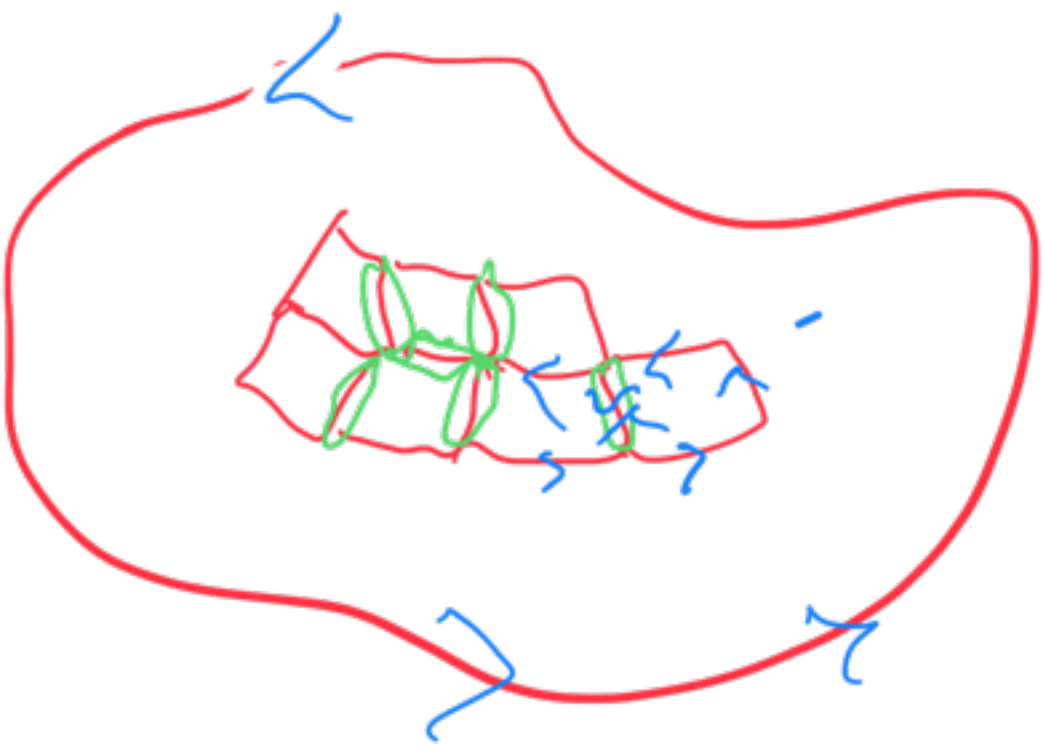
$= \epsilon_{ij1}\partial_j y$

$= \epsilon_{i21}$

$(\nabla \times A)_i \sim \sim \partial_i 3$

$= B$

$\vec{B} = - \hat{z}$



$$-\Delta \frac{1}{|r-x|} = \delta^{(3)}(\vec{x})$$

function

$x \longrightarrow x'$

$x \longrightarrow f(x) = y$

functional

$f \longrightarrow f(x)$

for a delta function

$\delta(x)$

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$$\frac{\partial f}{\partial x} \equiv \partial_x f$$

$$\partial_y f = \frac{\partial f}{\partial y}$$

$$\nabla \cdot B = 0$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$x'^i = x'^i(x^1, x^2, \ldots x^n)$$

$$V_i = g_{ij} V^j$$

$$V^k = g^{kl} V_l$$

$$y_{ik} \longrightarrow \eta_{ik}$$

$$\eta = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\frac{e^{i2\pi(n-m)} - e^{0}}{2\pi} = 0 \quad n \neq m$$

$$\int_0^{2\pi} \frac{e^{0}}{2\pi}\, dx = 1$$

$$\langle \vec{p} | \psi \rangle = \int \frac{d^3x}{(2\pi)^{3/2}} e^{i\vec{p}\cdot\vec{x}} \psi(\vec{x})$$

$$= \int \frac{d^3x}{(2\pi)^{3/2}} e^{i\vec{p}\cdot\vec{x}} \langle \vec{x} | \psi \rangle$$

$$\theta(x) = \begin{cases} 1 & x > 0 \\ 1/2 & x = 0 \\ 0 & x < 0 \end{cases}$$

$$\frac{d\theta(x)}{dx} = \delta(x).$$

$$\int_{-1}^{1} f(x)\delta(x)\,dx = f(0)$$

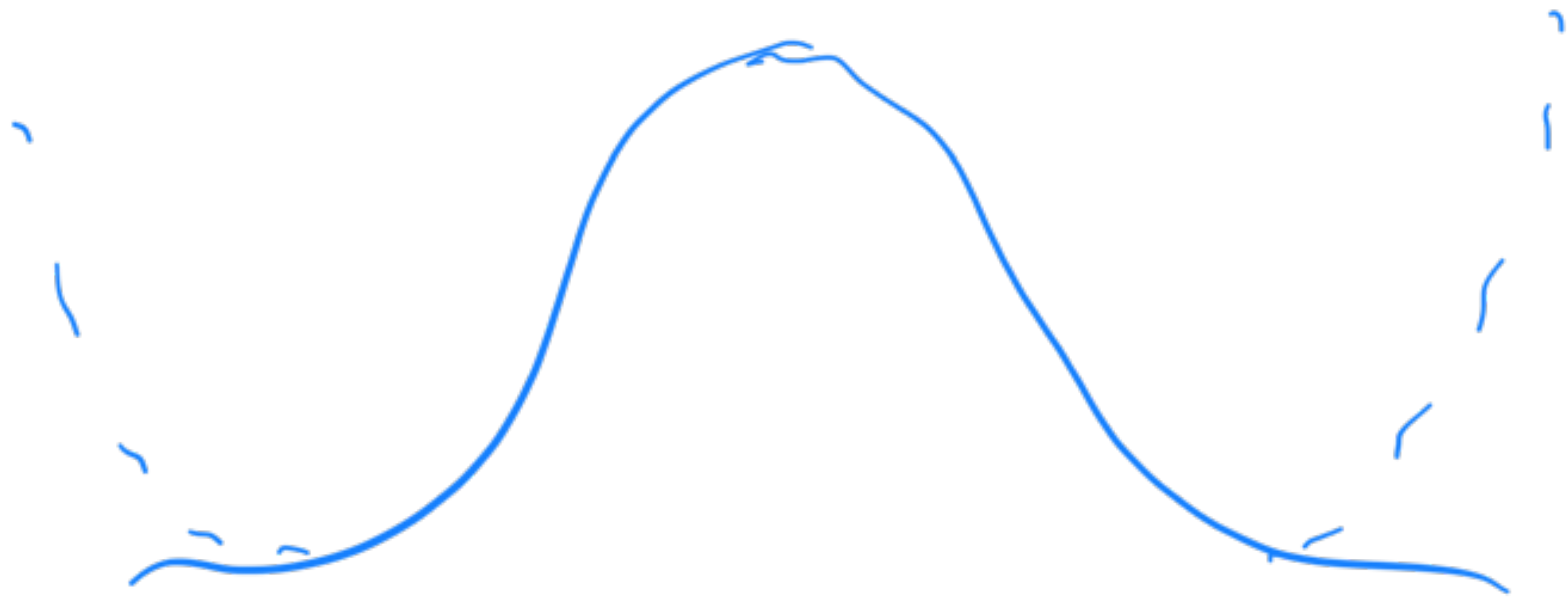
$$= \int_{-1}^{1} f(x)\frac{d\theta}{dx}\,dx$$

$$= \int_{-1}^{1} -f'(x)\theta(x)\,dx$$

keep surface term

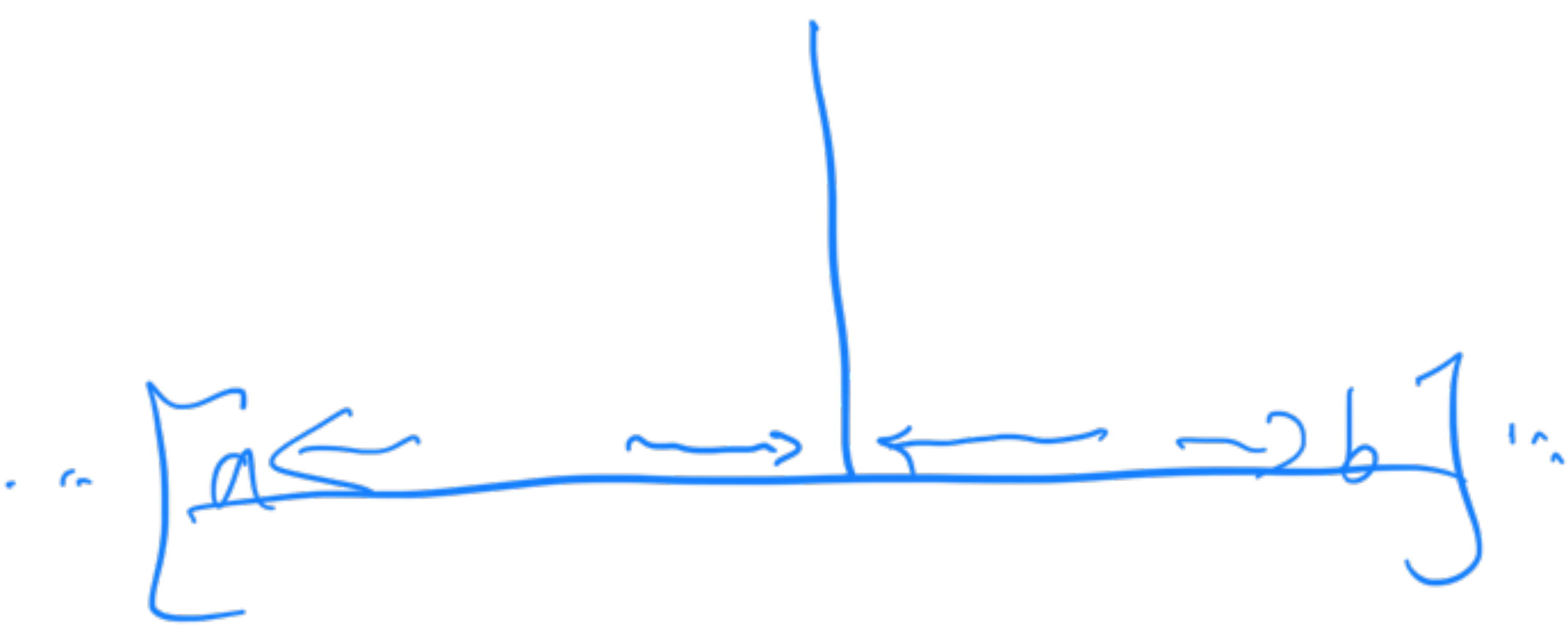
$$= \int_{0}^{1} -f'(x)\,dx \quad f(1)$$

$$= -f(1) + f(0)$$

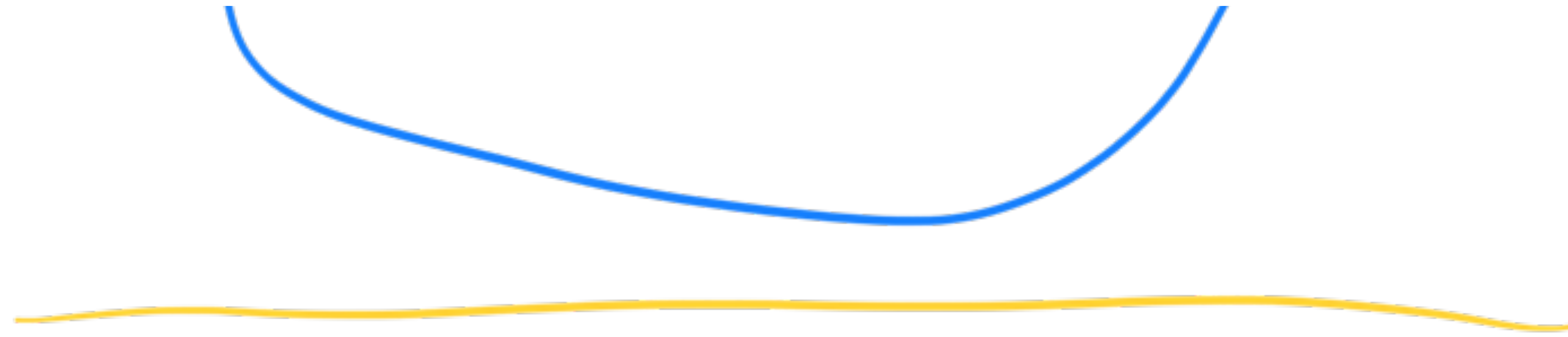


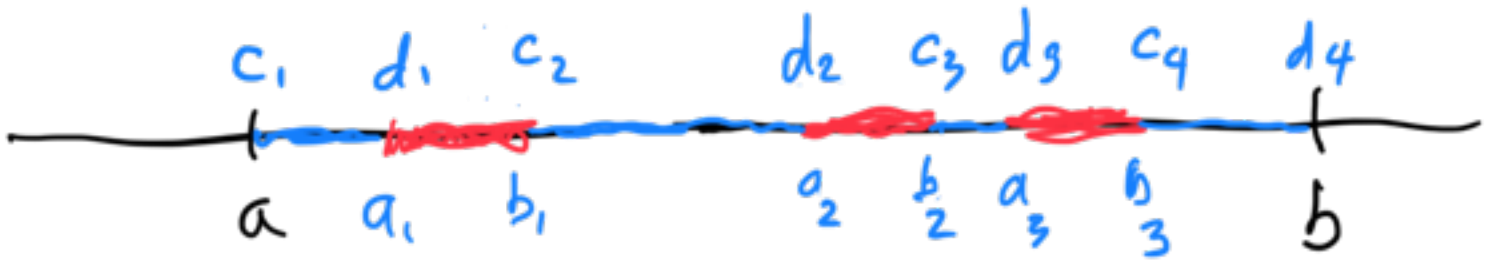
$\sum |f_n|$

$f(x)$



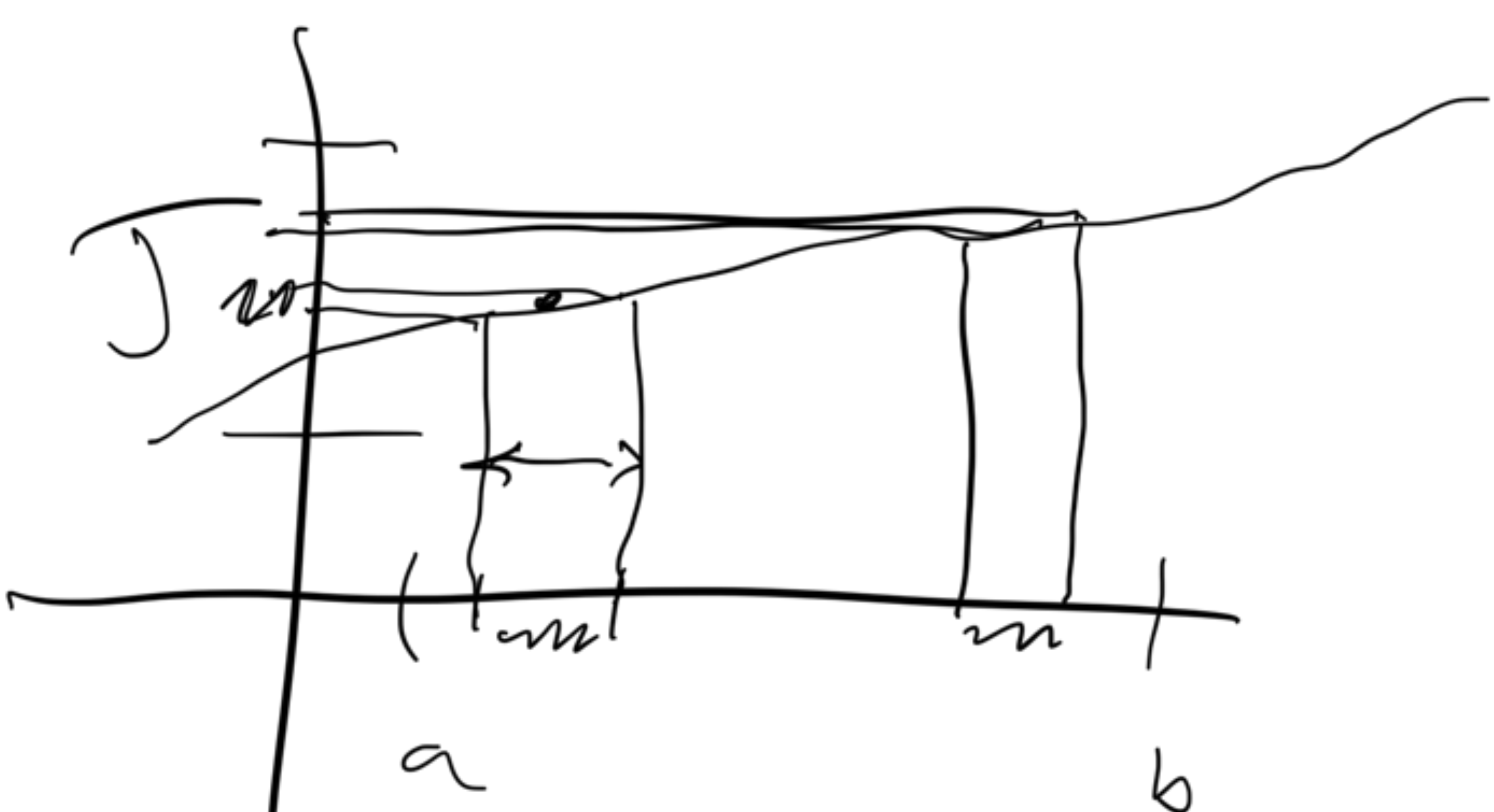
$f_n \sim \frac{1}{n^k}$



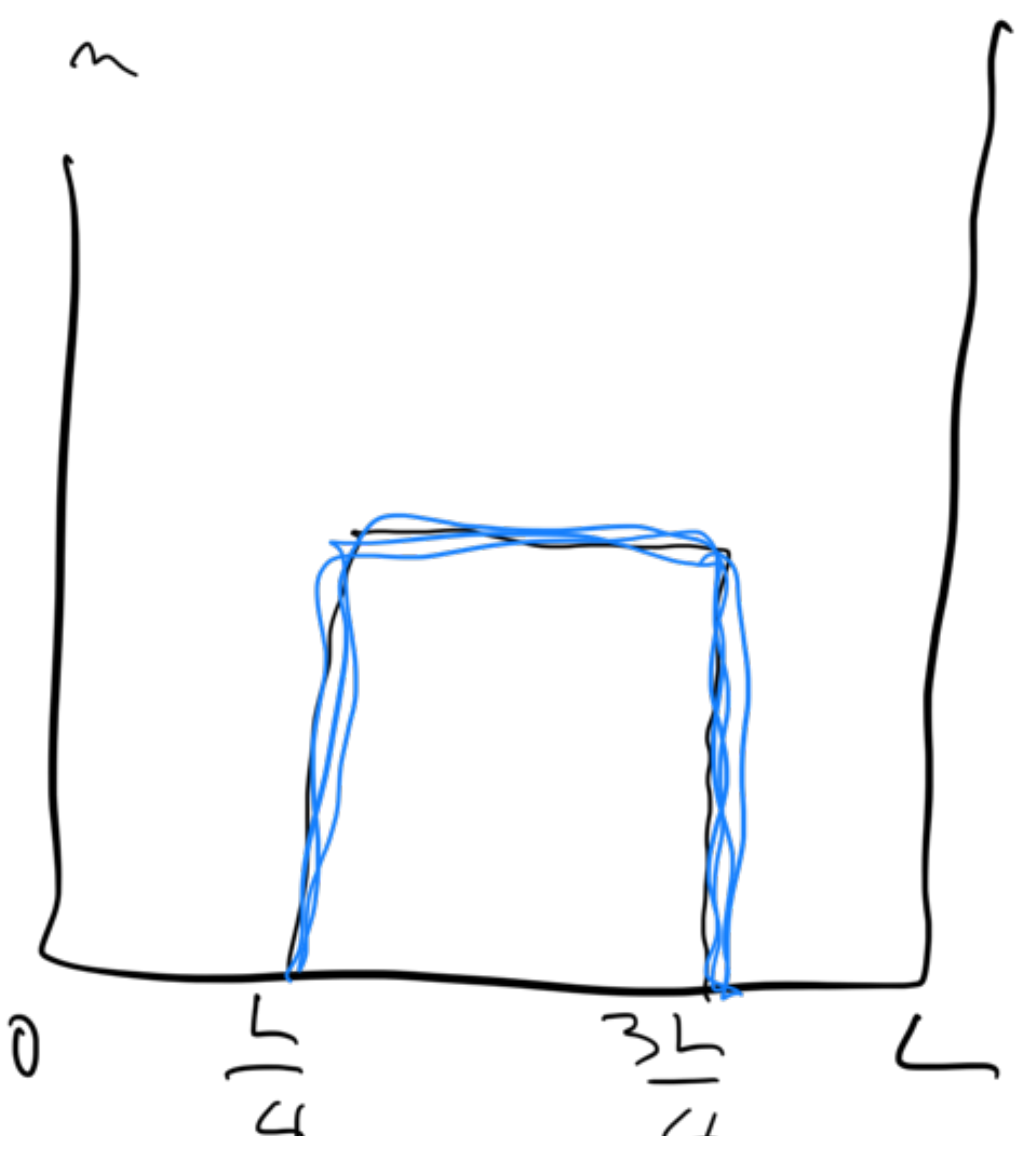


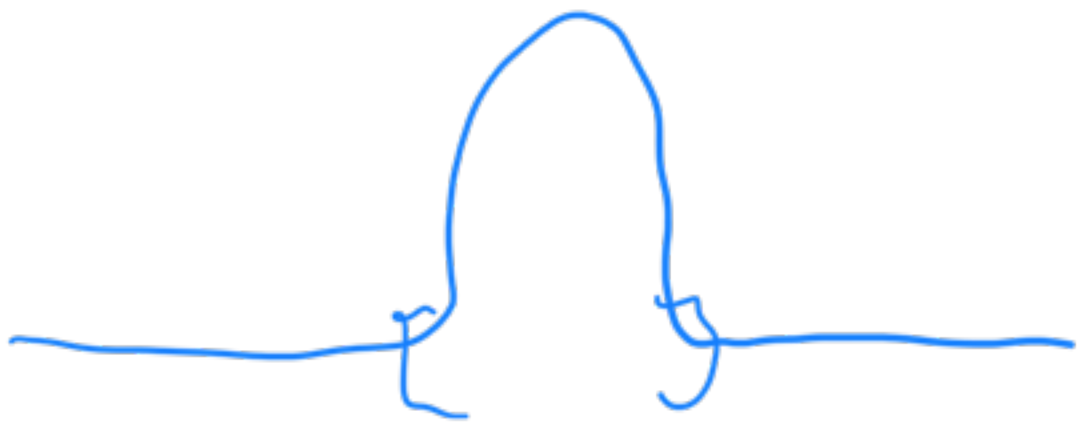
$$\int_a^b f(x)\,\underline{dx} = \sum_i f(x_i)\,m(S_i)$$

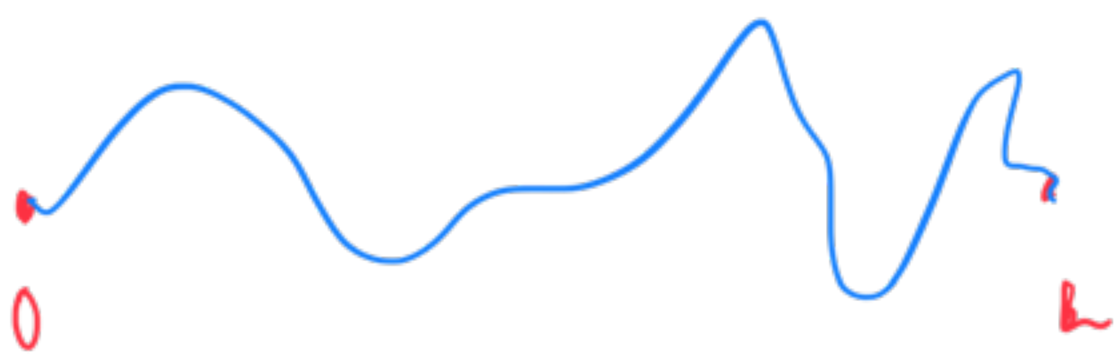
measure of set S_i of points f maps to interval J_i containing $f(x_i)$.

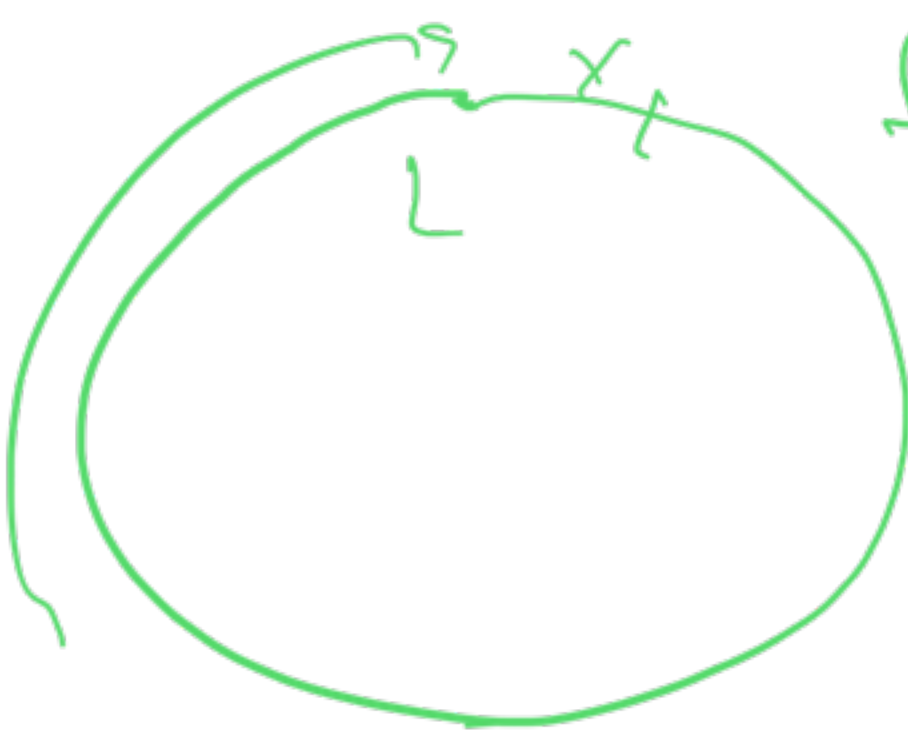


$$\psi(x,t)=\sum_n e^{-iE_nt/\hbar}\,\psi_n(x)$$









$$f(L) = f(0)$$

$$f(x+L) = f(x)$$

$$e^{\frac{iL\,2\pi\hbar\,3}{L\hbar}} = e^{i3\cdot 2\pi} = 1$$

$\theta(x)$

$$\theta(x) = \begin{cases} 1 & x > 0 \end{cases}$$

$\theta(x)\ \{0 \quad x<0$

$\theta'(x) = \delta(x)$

$$\int_{-1}^{1} (f(x)\theta(x))' dx$$

$$= f(1)\theta(1) - f(-1)\theta(-1) = f(1)$$

$$= \int_{-1}^{1} f(x)\theta'(x)dx$$

$$+ \int_{-1}^{1} f'(x)\theta(x)dx$$

$$= \int_{-1}^{1} f(x)\theta'(x)dx + \int_{0}^{1} f'(x)dx$$

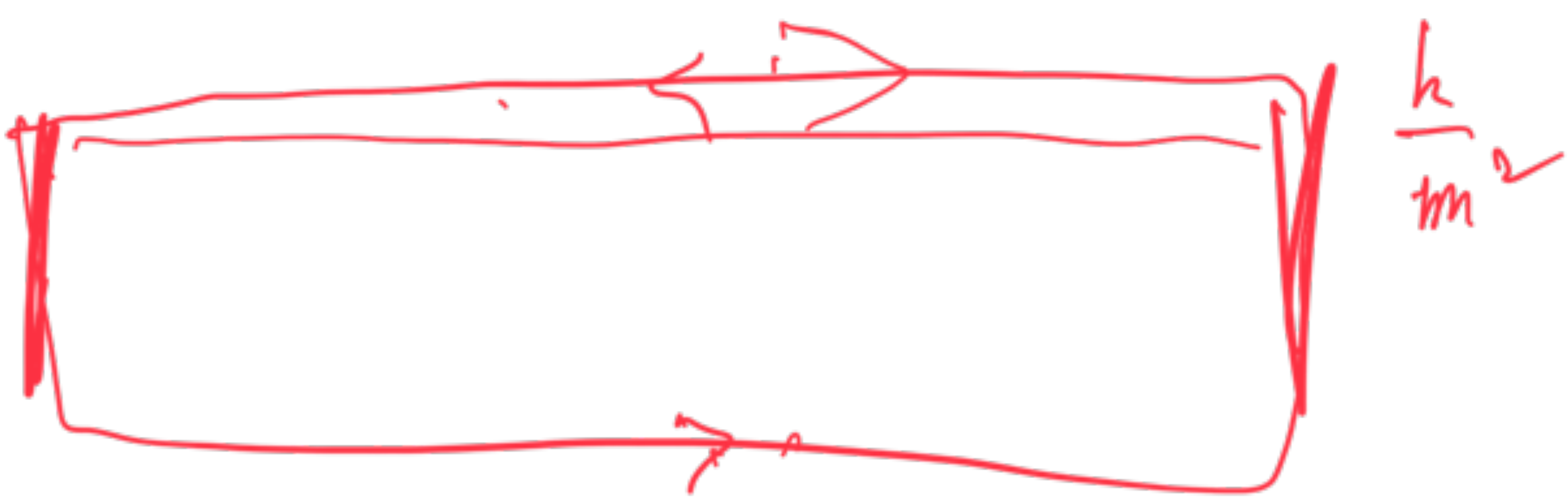
$$= \int_{-1}^{1} f(x)\theta'(x)dx + f(1) - f(0)$$

$$f(1) = f(1) - f(0) + \int_{-1}^{1} f(x)\theta'(x)dx$$

$$f(0) = \int^{1} f(x)\,\theta'(x)dx$$

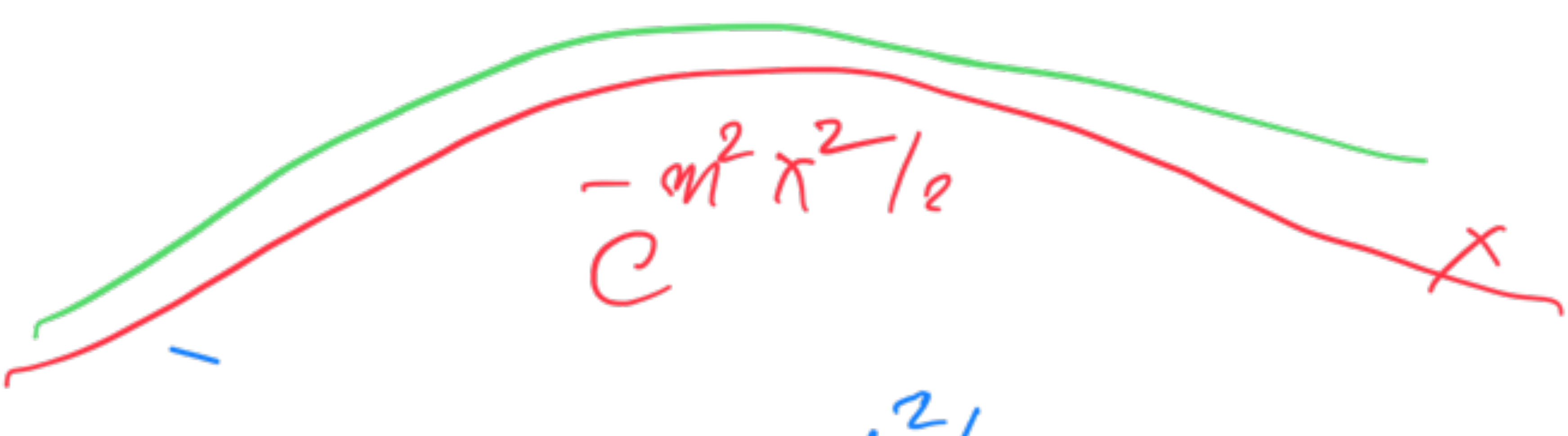
$$= \int_{-1}^{1} f(x)\,\delta(x)\,dx$$

$$= f(0).$$



$$z = x + iy$$

$$y = \frac{k}{m^2}$$



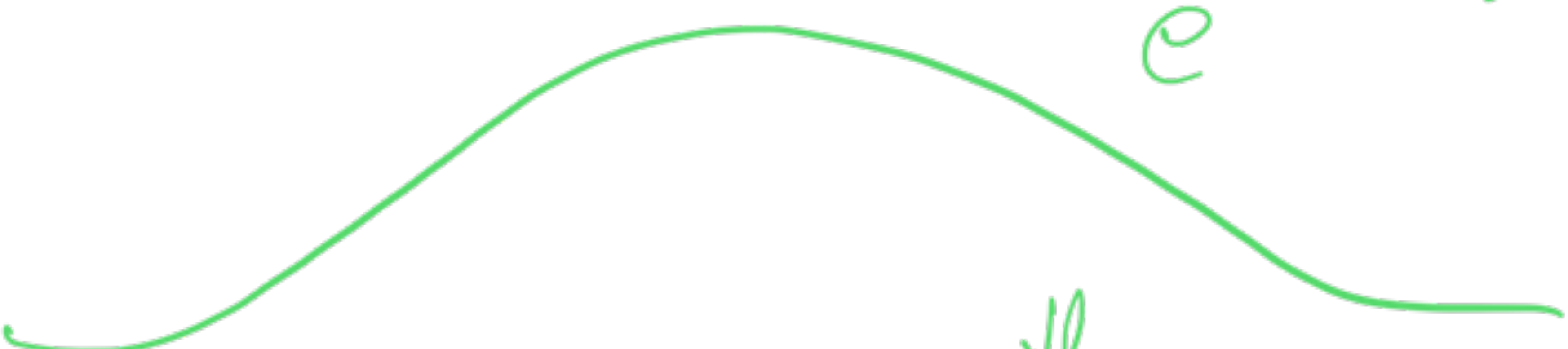
sharp

$e^{-m^2 x^2/2}$

big m

sharp

$e^{-k^2/2m^2}$



smooth

$$\delta(x-y) = \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ik(x-y)}$$

$$f(x) = \int f(y)\,\delta(x-y)\,dy$$

$$= \int_{-\infty}^{\infty} f(y)\,dy \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ik(x-y)}$$

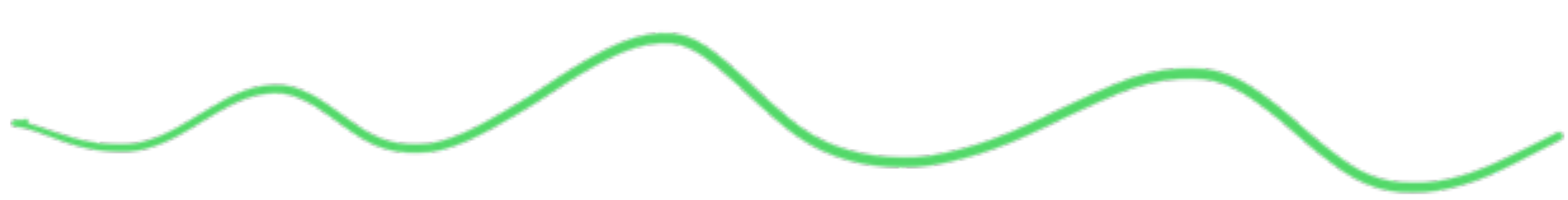
$$\boxed{= \int \frac{dy}{\sqrt{2\pi}} e^{+ikx} \int \frac{dk}{\sqrt{2\pi}} e^{-iky} f(y)}$$

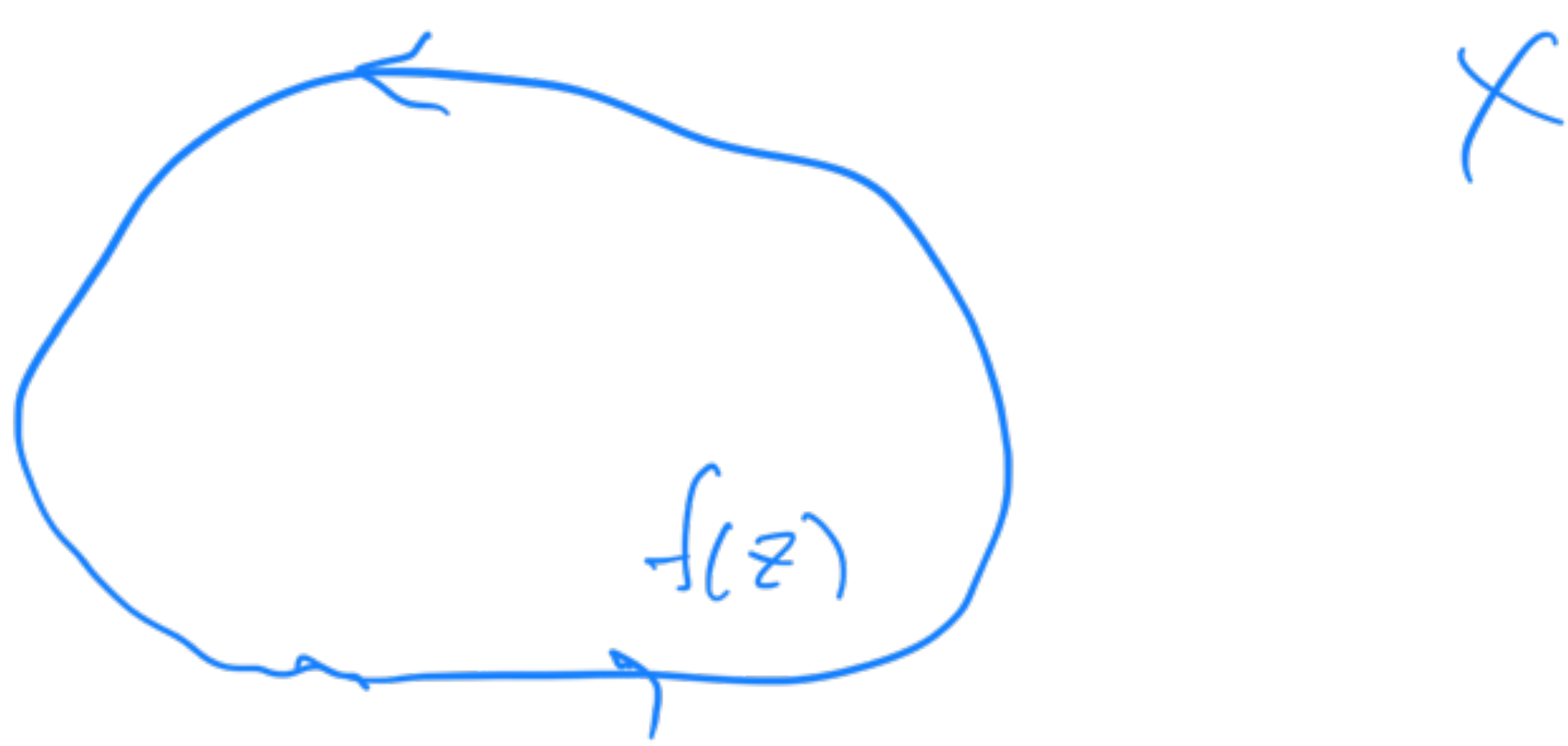
$$= \int \frac{dk}{\sqrt{2\pi}} \tilde{f}(k) e^{ikx}$$

$$\tilde{f}(k) = \int \frac{dy}{\sqrt{2\pi}} e^{-iky} f(y)$$

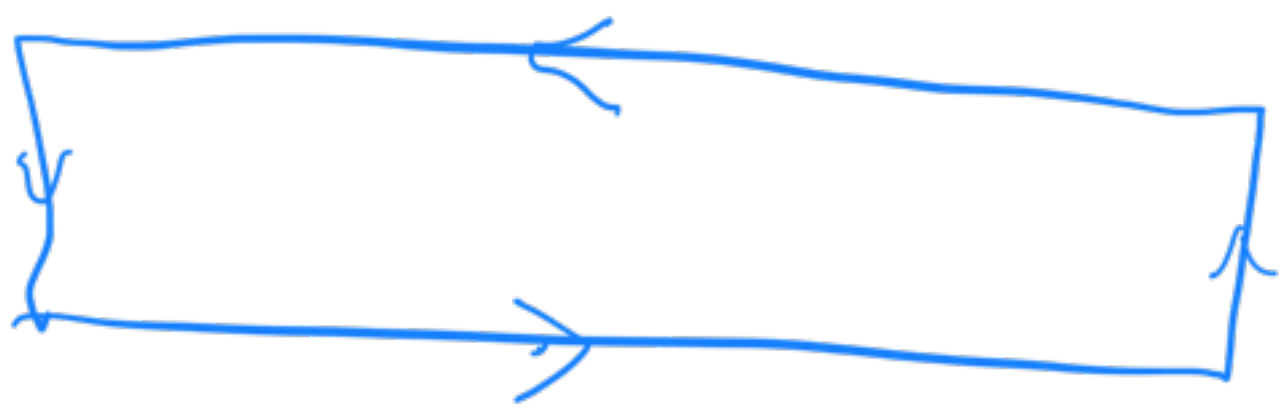
$$[\phi] = n$$

if $|\phi - n| < \frac{1}{2}$

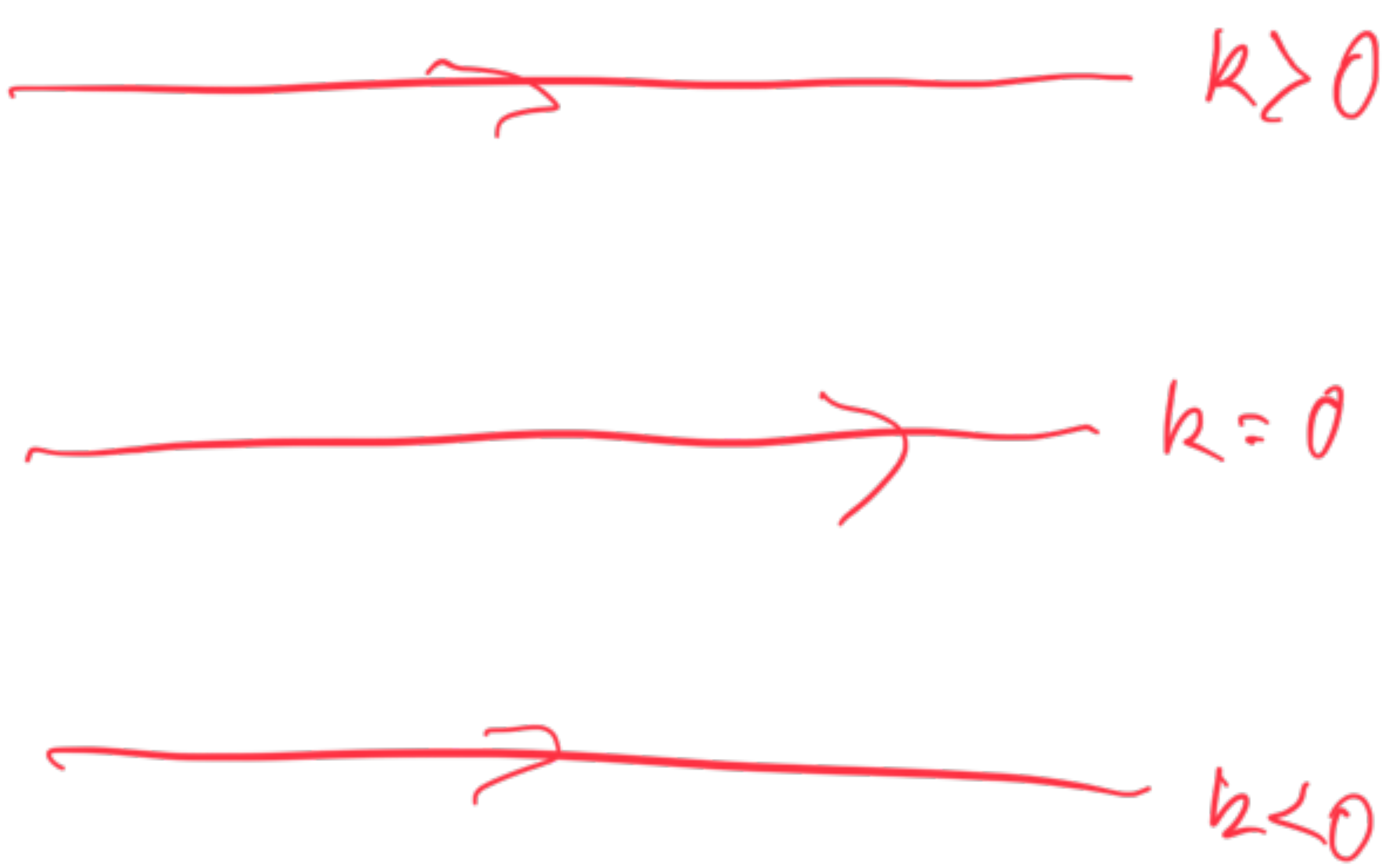




$$\oint f(z)\,dz = 0$$



$$\int_{\square} = 0 \qquad \int_{\rightarrow} = \int_{\leftarrow}$$



$$\delta_N(x) = \frac{\sin Nx}{\pi x}$$

$$\phi(x) = \int \frac{dk}{\sqrt{2\pi\omega_k}} \left[a(k) e^{ik\cdot x} + a^\dagger(k) e^{-ik\cdot x} \right]$$

$$[a(k), a^\dagger(p)] = \delta^{(3)}(k-p)$$

$$= a(k)a^\dagger(p) - a^\dagger(p)a(k)$$

$$\{a(k,s), a^\dagger(p,s')\}$$

$$= a(k,s)\,a^\dagger(p,s')$$

$$+ a^\dagger(p,s')\,a(k,s) = \delta_{ss'}\,\delta(k-p)$$

$$\psi(x) = \sum_s \int dk\; a(k,s)\,e^{ik\cdot x}\,\epsilon(k,s)$$

$$+ a_c^\dagger(k,s)\,e^{-ik\cdot x}\,\epsilon^*(k,s)$$

$-$ el. $\qquad\qquad$ $+$ pos

$$\mathrm{Tr}\left[\rho\left(A-\langle A\rangle\right)^2\right]$$

$$= \mathrm{Tr}\,\rho A^2 - 2\,\mathrm{Tr}\,\rho A\langle A\rangle$$

$$+ \mathrm{Tr}\,\rho\langle A\rangle^2 \qquad -\cancel{\mathrm{Tr}\,\rho}\langle A\rangle^2$$

$$\rho \geq 0 \qquad \langle s|\rho|s\rangle \geq 0$$

$$\mathrm{Tr}(\rho) = \sum_n \langle n|\rho|n\rangle = 1$$

$$\rho = \rho^\dagger$$

$$= \sum |n\rangle\langle n|\,\rho_n$$

$$\langle (A-\langle A\rangle)^2\rangle = \langle A^2\rangle - \langle A\rangle^2$$

$$f(x) = \int \frac{d^n k}{(2\pi)^{n/2}} e^{ik\cdot x} \tilde{f}(k)$$

$x = x_1, x_2 \cdots$

$k = k_1, k_2, \cdots$

---

coherent state $|\alpha\rangle$

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

$$e^{-\xi^* a}|\alpha\rangle = e^{-\xi^*\alpha}$$

$$\langle\alpha| a^\dagger = \langle\alpha|\alpha^* = \alpha^*\langle\alpha|$$

$$\langle\alpha| e^{\xi a^\dagger} = \langle\alpha| e^{\xi\alpha^*}$$

$$\chi(\alpha) = \mathrm{Tr}\left(e^{\xi a^\dagger} e^{-\xi^* a}\right)$$

$$\Lambda_\alpha(x) = \mathrm{Tr}\left(e^{-\xi a^*} \rho\, e^{\xi a^\dagger}\right)$$

$$\nabla_x e^{i\vec{k}\cdot\vec{x}} = i\vec{k}\cdot e^{ik\cdot x}$$

$$\nabla^2 e^{ik\cdot x} = -\vec{k}^2 e^{ik\cdot x}$$

$$-\nabla^2 G(x-x') = \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2}{\vec{k}^2} e^{i\vec{k}\cdot(\vec{x}-\vec{x}')}$$

$$= \delta^3(\vec{x}-\vec{x}')$$

$$E = -\nabla\phi + \nabla\times A$$

$$\nabla\cdot A = 0$$

Coulomb-gauge condition

$$A(\underline{\vec{x}+\vec{a}}) = \frac{\mu_0}{4\pi}\int d^3x' \frac{J(\vec{x}')}{|\vec{x}+\vec{a}-x'|}$$

$$\frac{\mu_0}{4\pi}\int d^3x' \; J(\vec{x}'+\vec{a})$$

$$= \frac{\mu_0}{4\pi}\int d^3x' \frac{\quad}{|x - x'|}$$

$$\widetilde{f*g}(k) = \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} e^{-ikx} \quad f*g(x)$$

8 Sep.

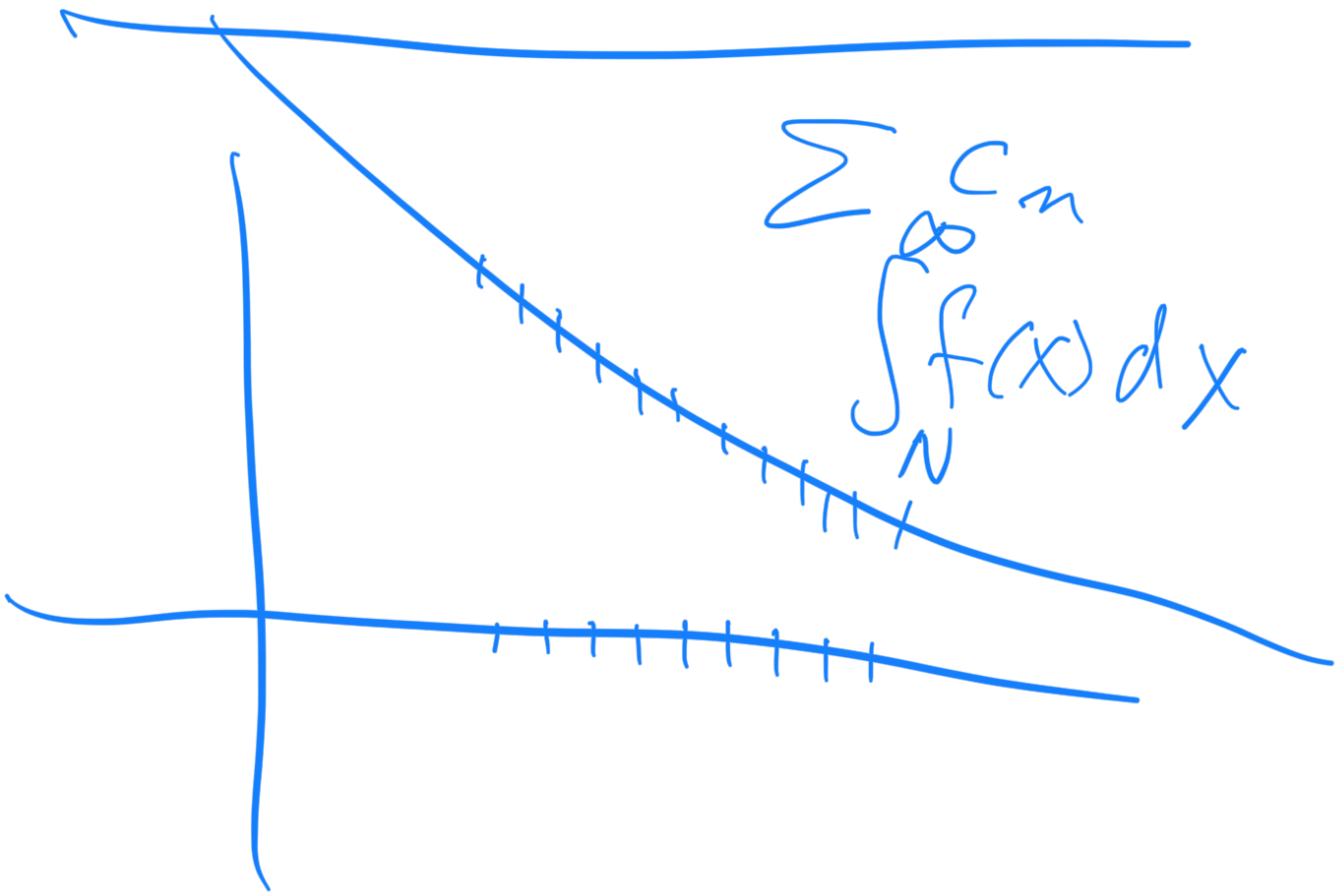
$$\int x\,\delta(x)\,dx = 0$$

$$\int f(x)\,\delta(x)\,dx = f(0)$$

$$\int e^{ik\cdot x}\,\delta(P(ik))\,h(k)\,d^n k$$

$$= \int e^{ik\cdot x}\,\delta\big((k^0-\omega_k)(k^0+\omega_k)\big)\,h(k)\,\frac{d^4k}{(2\pi)^4}$$

$$= \int e^{ik\cdot x} \sum_{k^0=\pm\omega_k} \frac{h(k^0)}{|2\omega_k|}\,\frac{d^4k}{(2\pi)^4}$$



$$\sum c_n$$

$$\int_N^\infty f(x)\,dx$$

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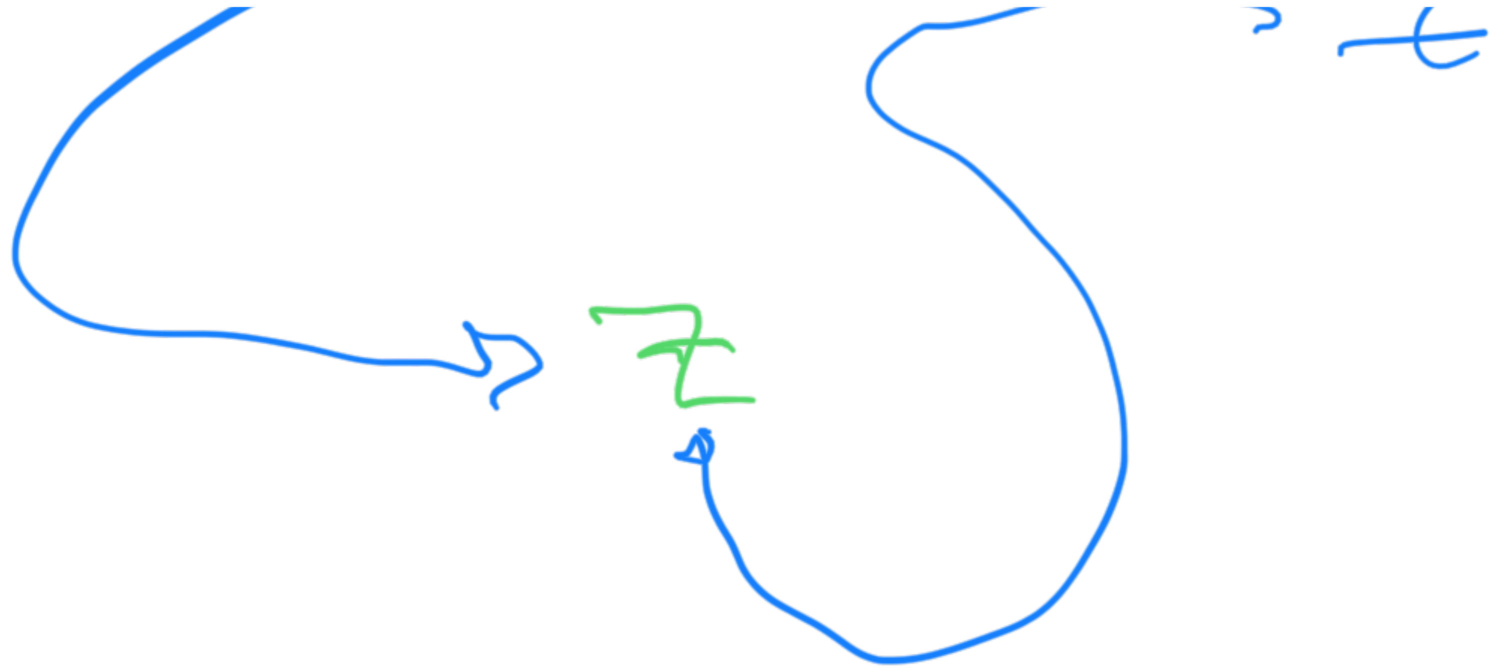
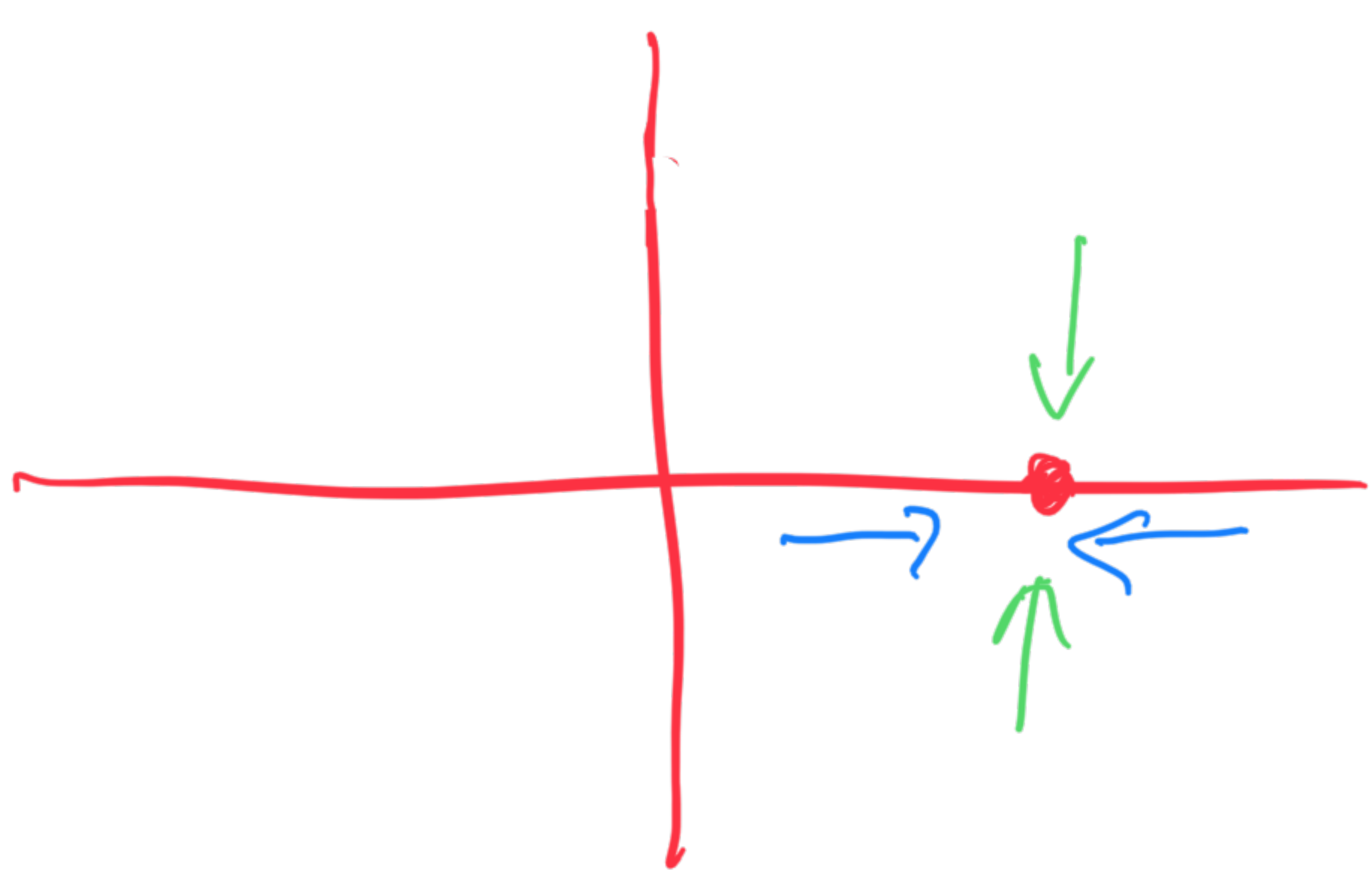
$$\frac{1}{1-x} = \sum x^n$$



$|z-z_0|<R$

$x' \longrightarrow x \longleftarrow x'$

$(1,0)$



Is $f(x,y)$ analytic?

$$f(x,y) = u(x,y) + i v(x,y)$$

$$\frac{df}{dz} = \frac{f' \, dz}{dz} = f'$$

$$g = (3, 2)$$

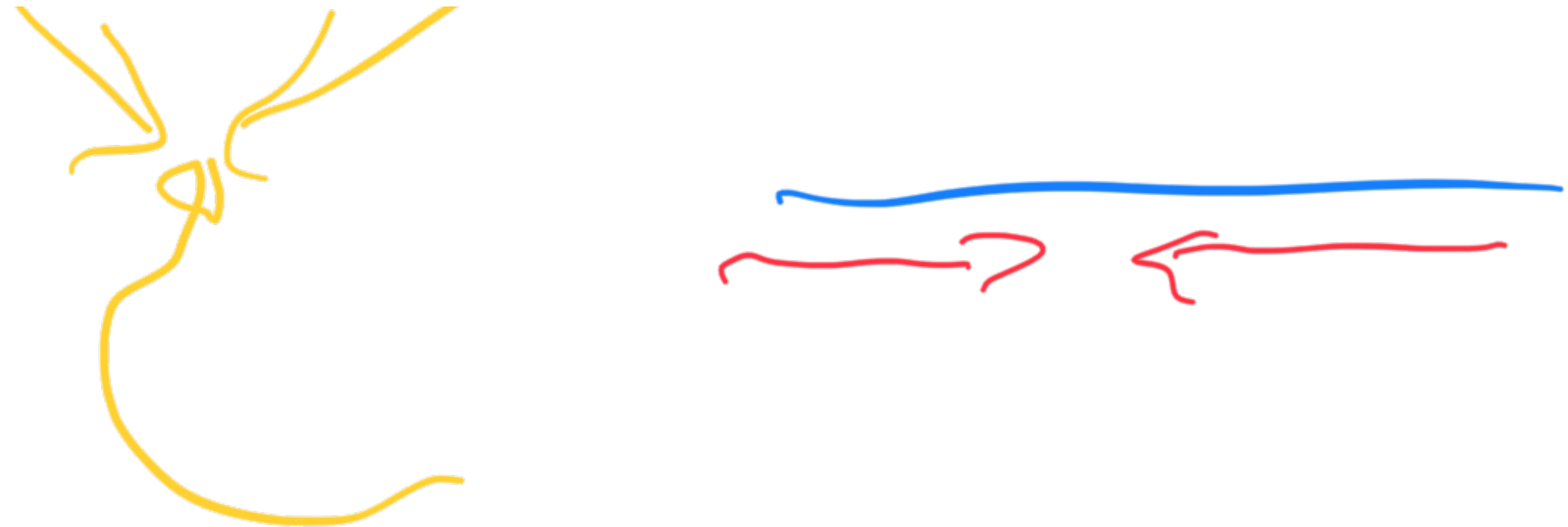
$$\operatorname{Re} g = 3 \qquad \operatorname{Im} g = 2$$

$$u_x = v_y \qquad v_x = -u_y$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

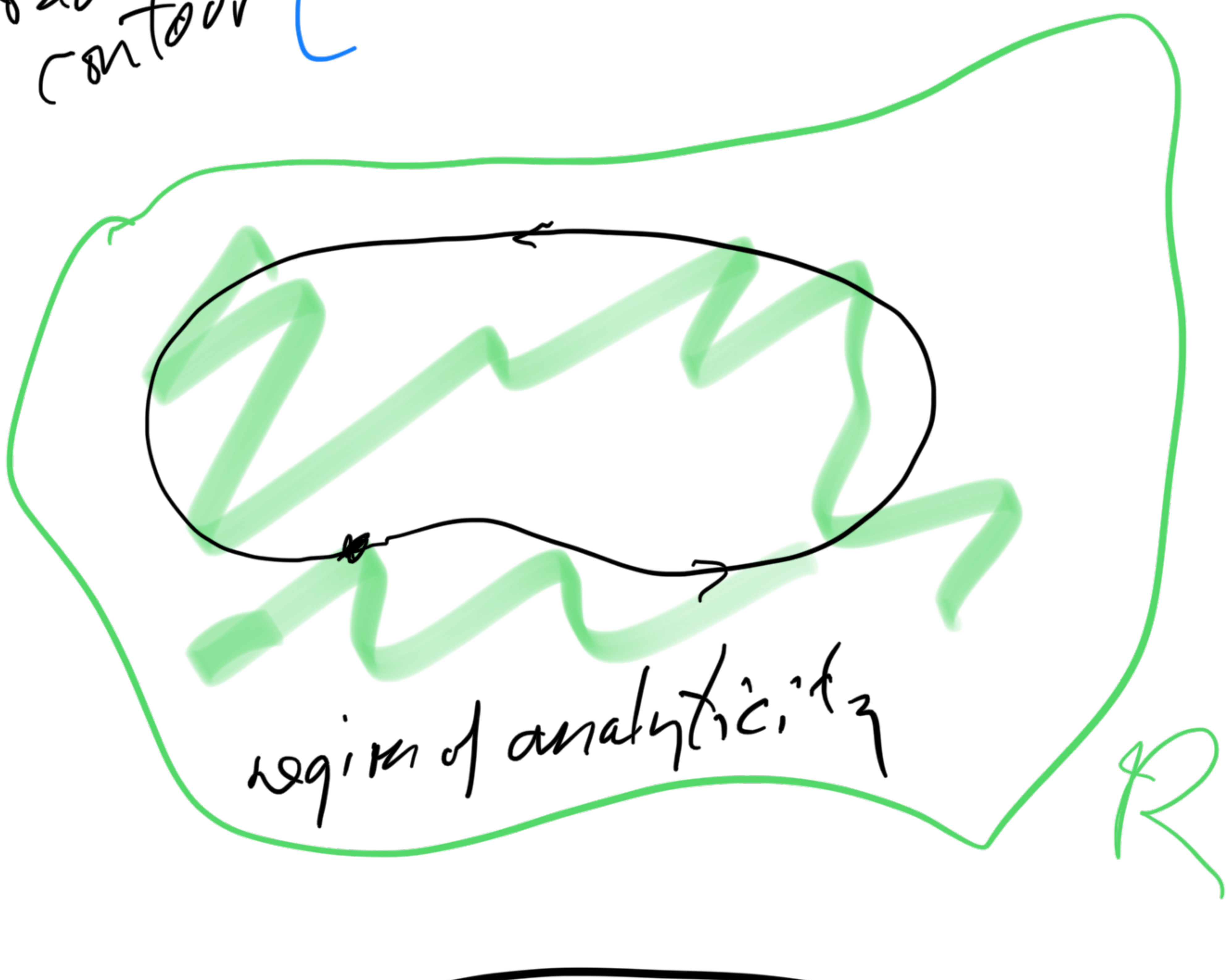
pde

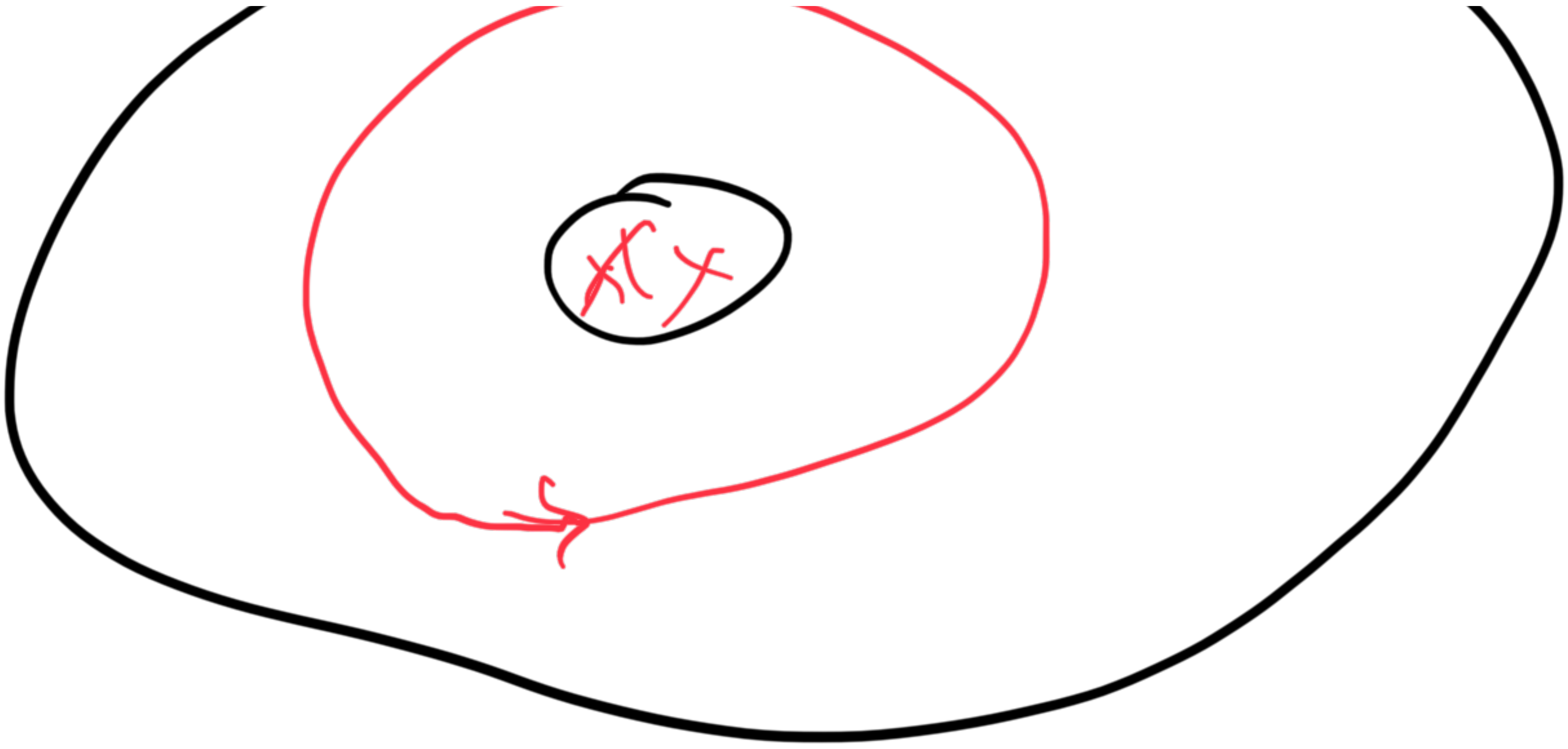
$$f = u + iv$$

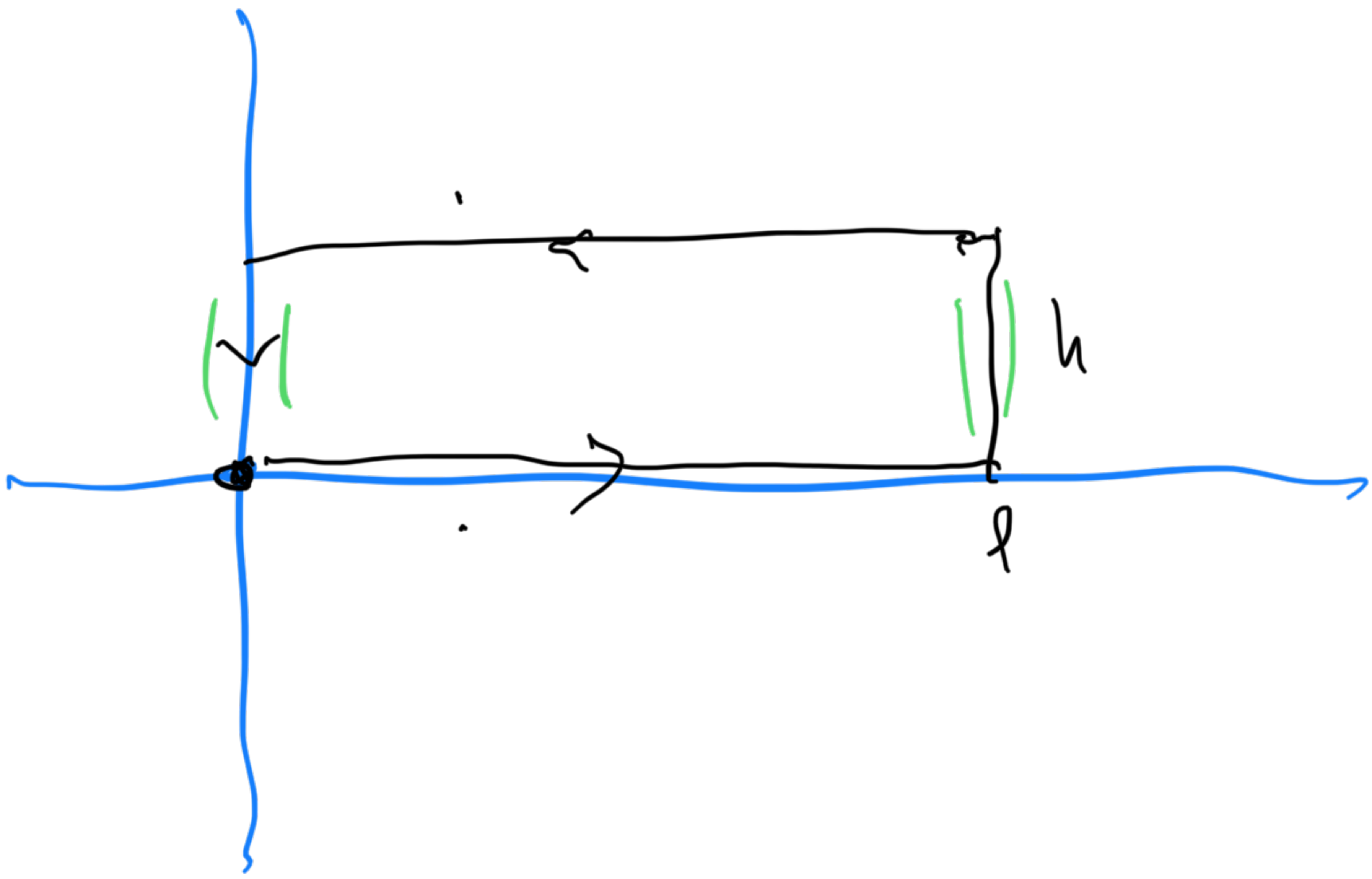


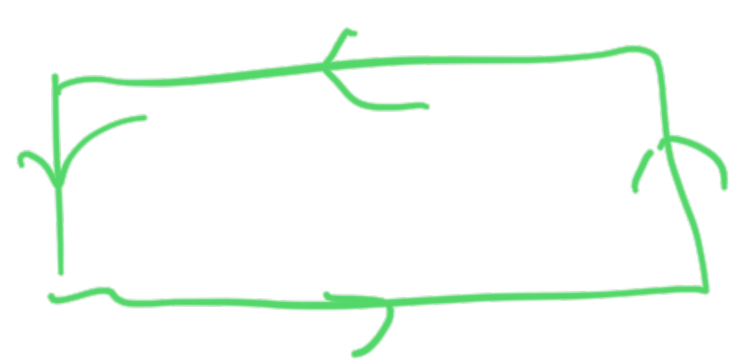
$$\oint_C f(z)\,dz = 0$$

closed contour









$$\oint_R f(z)\,dz = 0$$

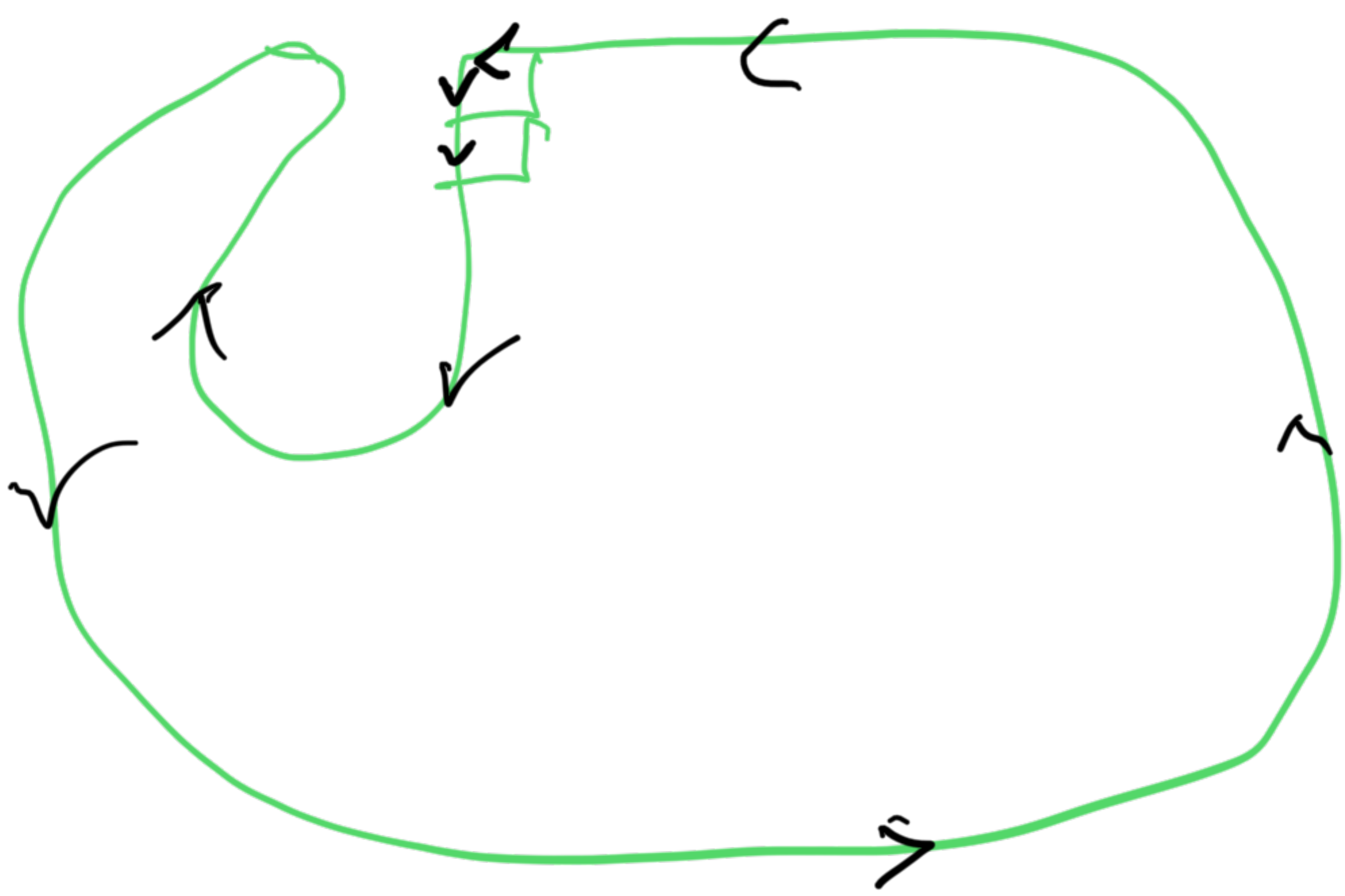
all R's i

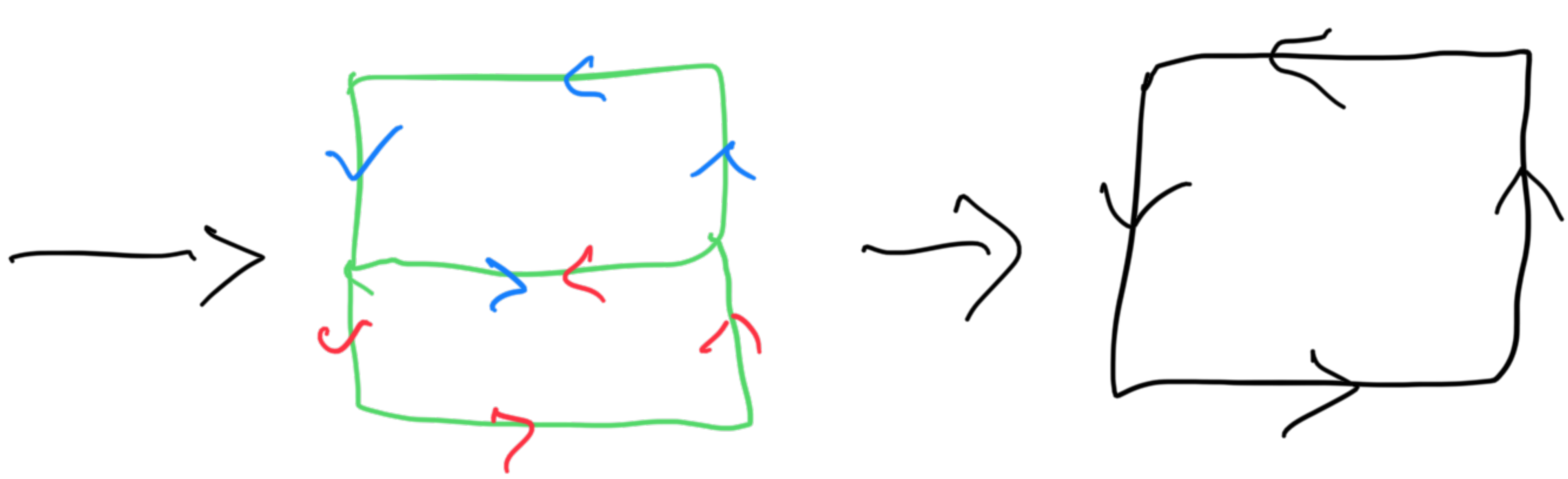
$f(z)$ is entire

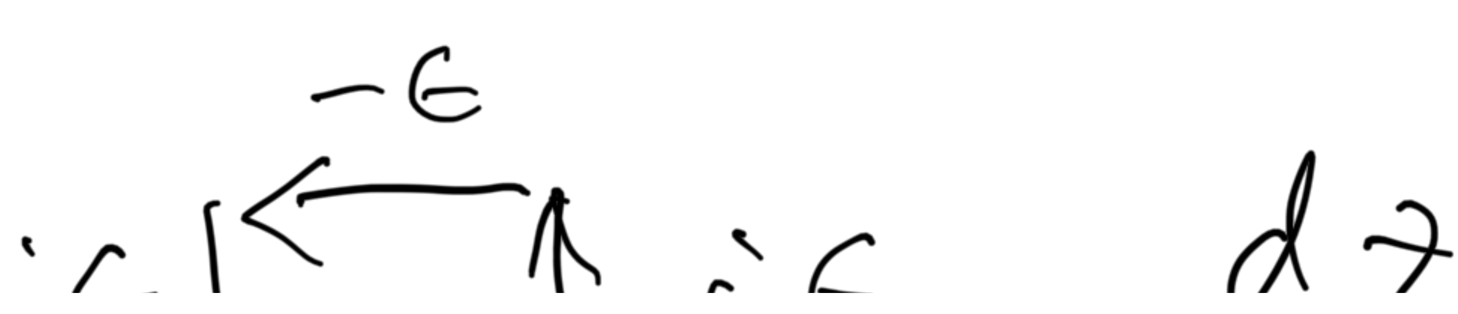
i.e., analytic everywhere

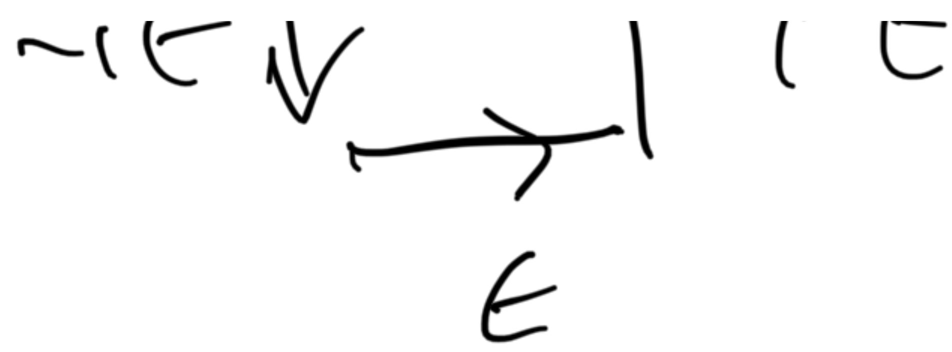
i.e., differentiable "

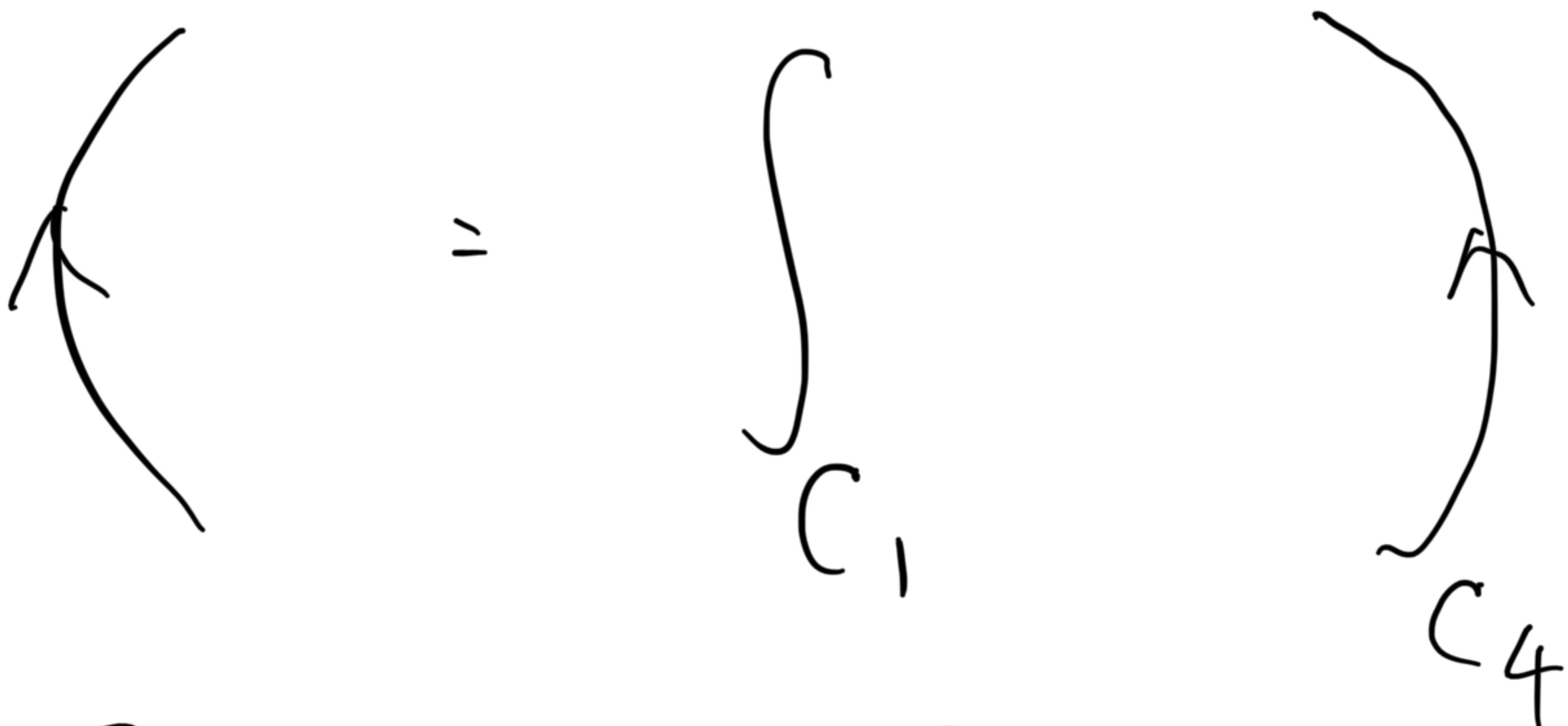
i.e., Cauchy-Riemann





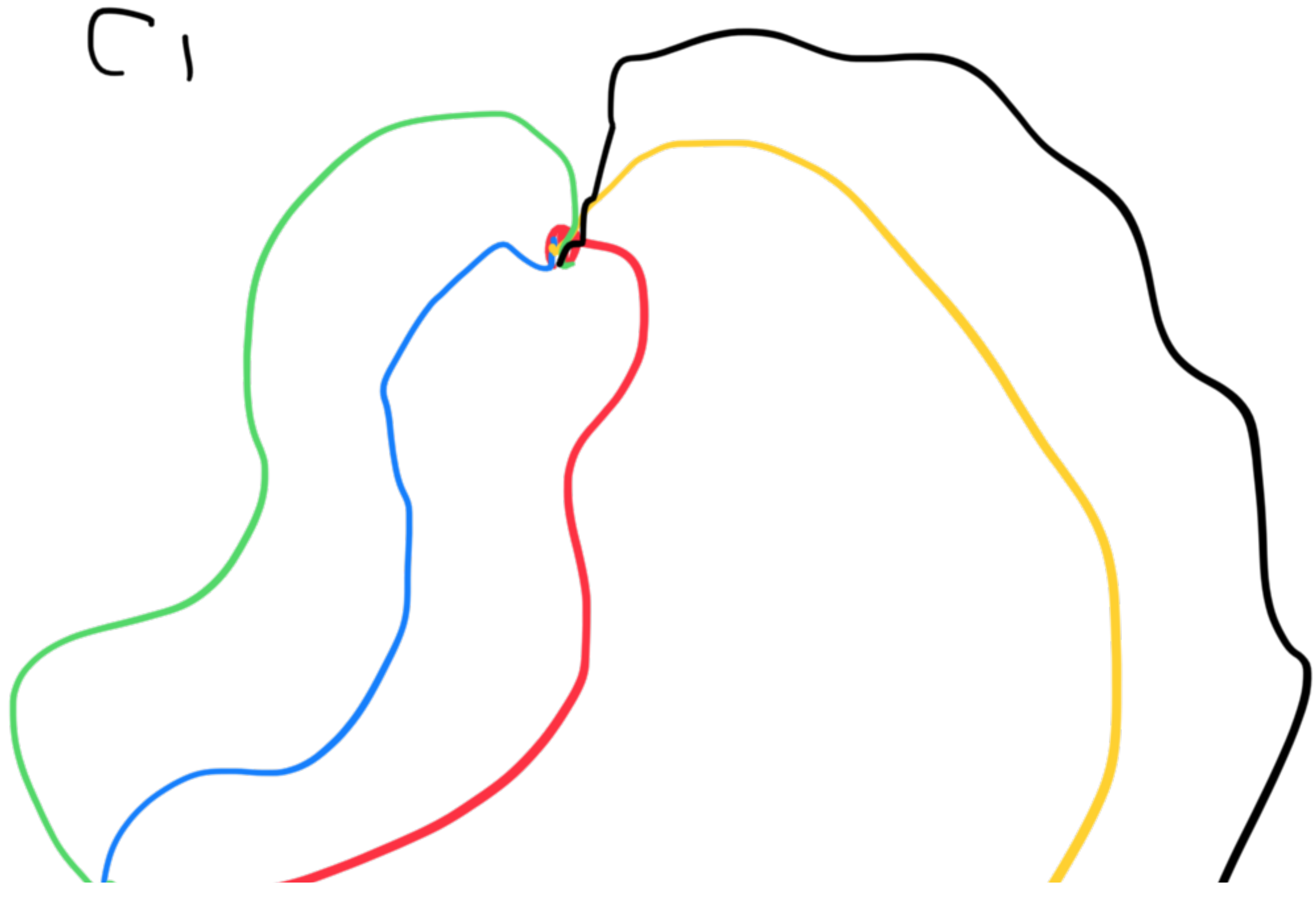


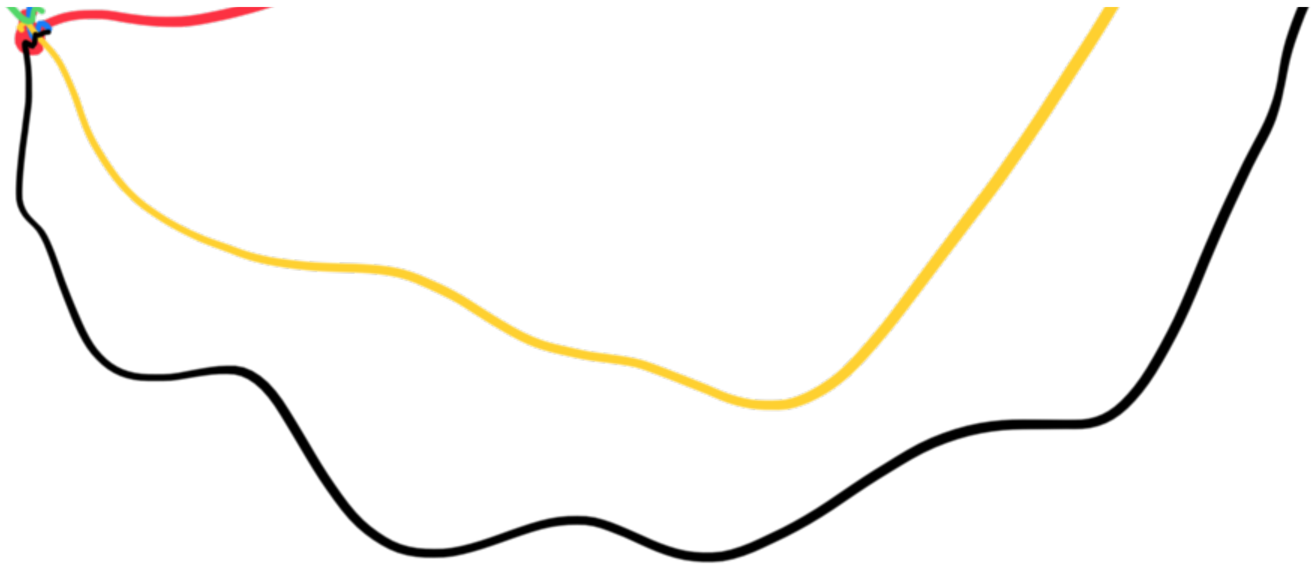




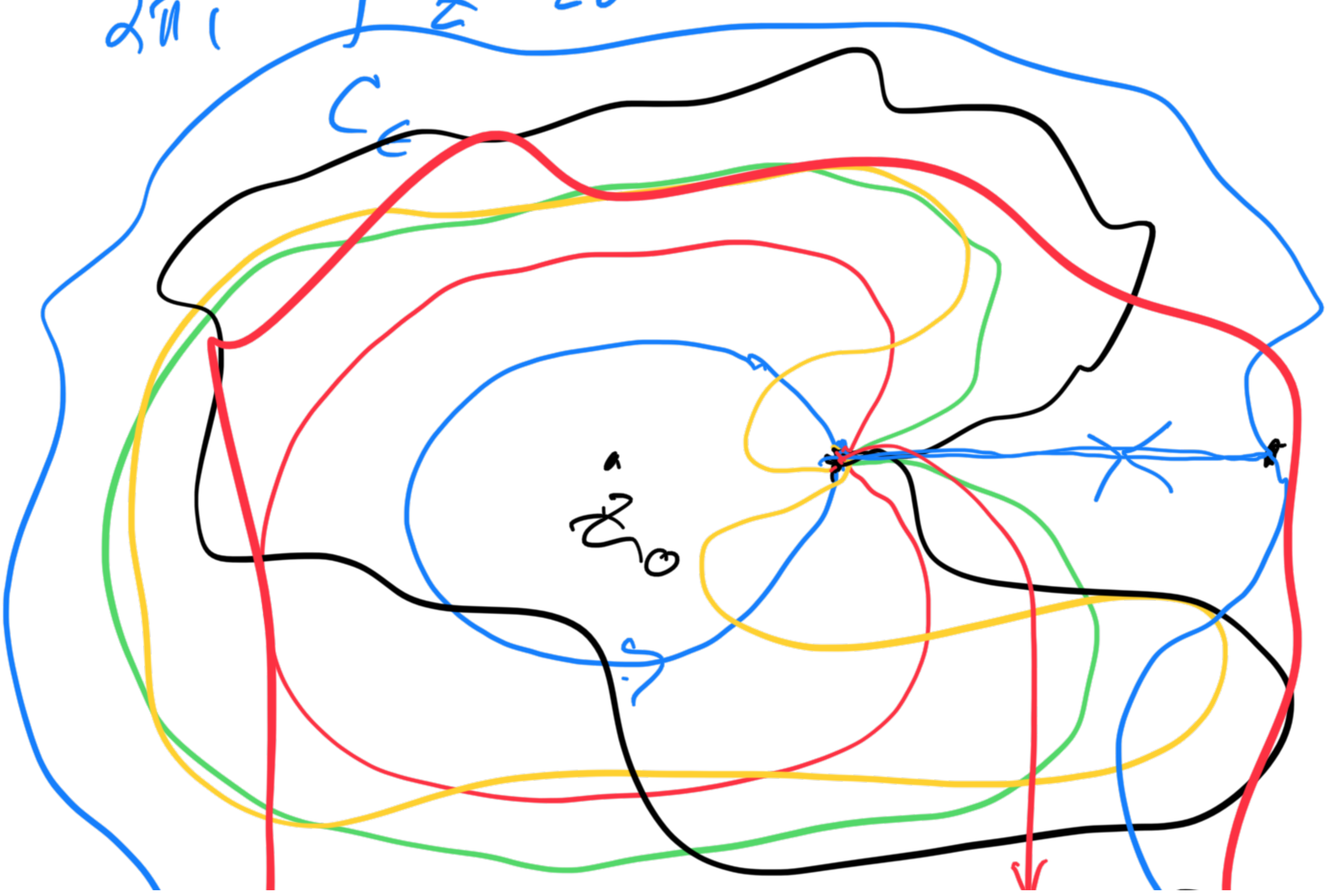
$$0 = \oint f(z)\,dz = \int_{C_4} f(z)\,dz$$

$$-\int_{C_1} f(z)\,dz = \oint f(z)\,dz$$





$$\frac{1}{2\pi i}\oint_{C_\epsilon} \frac{f(z)}{z-z_0}\,dz = f(z_0)$$



$$\frac{1}{2\pi i}\oint_C \frac{f(z)}{z-z_0}\,dz = f(z_0)$$

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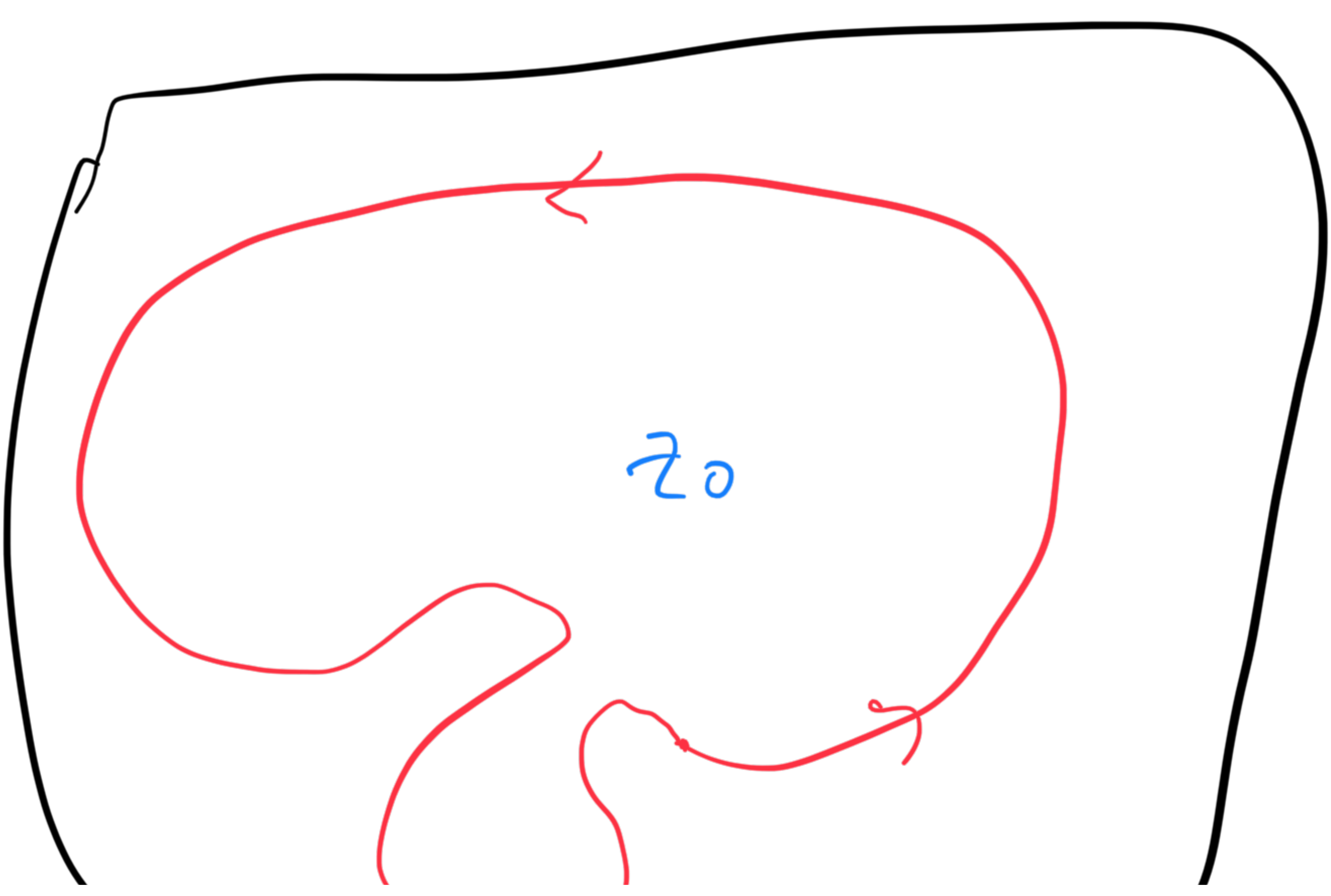

simply connected analyticity