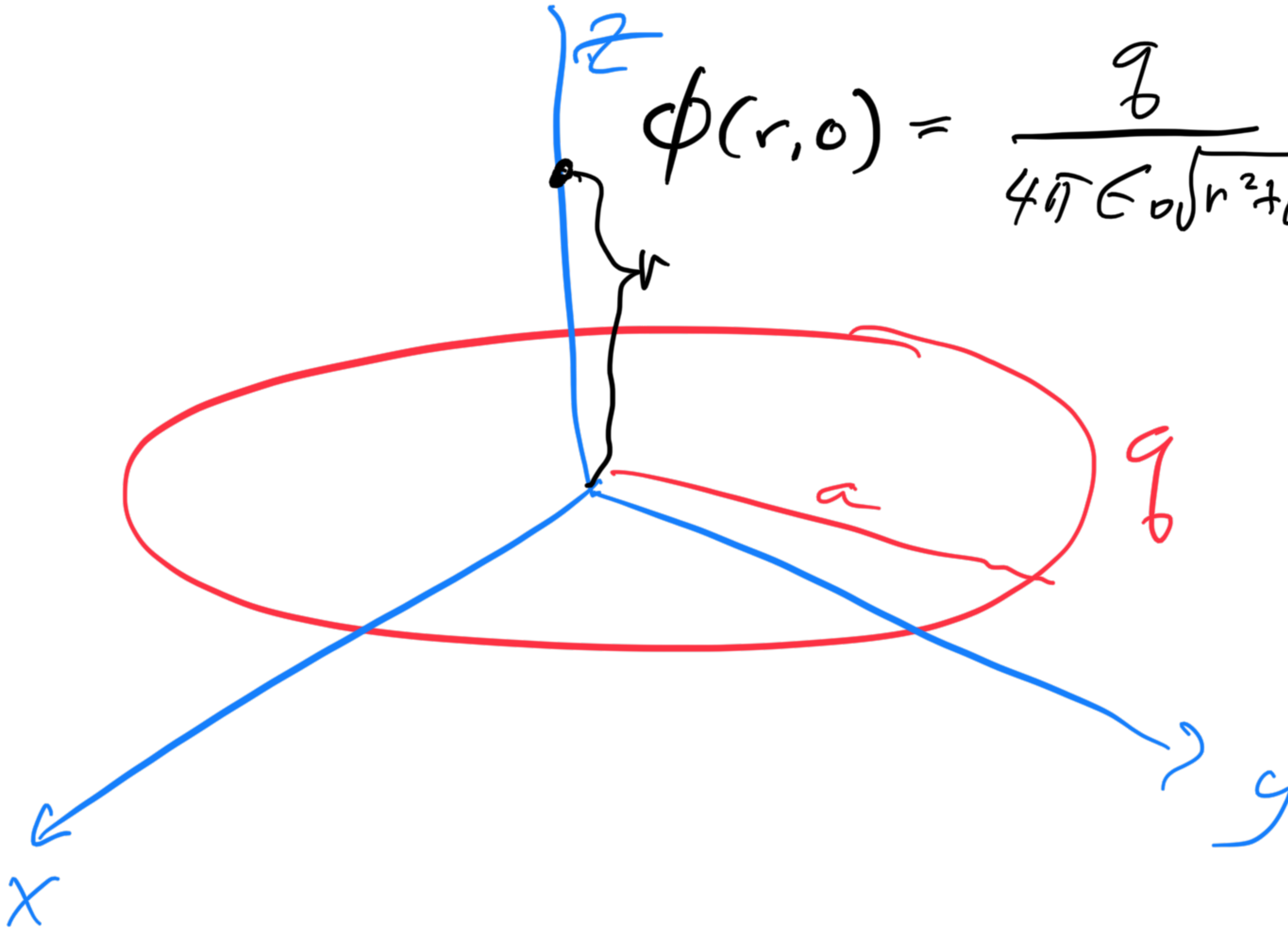
9 Nov.

$$\phi(r,0) = \frac{q}{4\pi\epsilon_0\sqrt{r^2+a^2}}$$



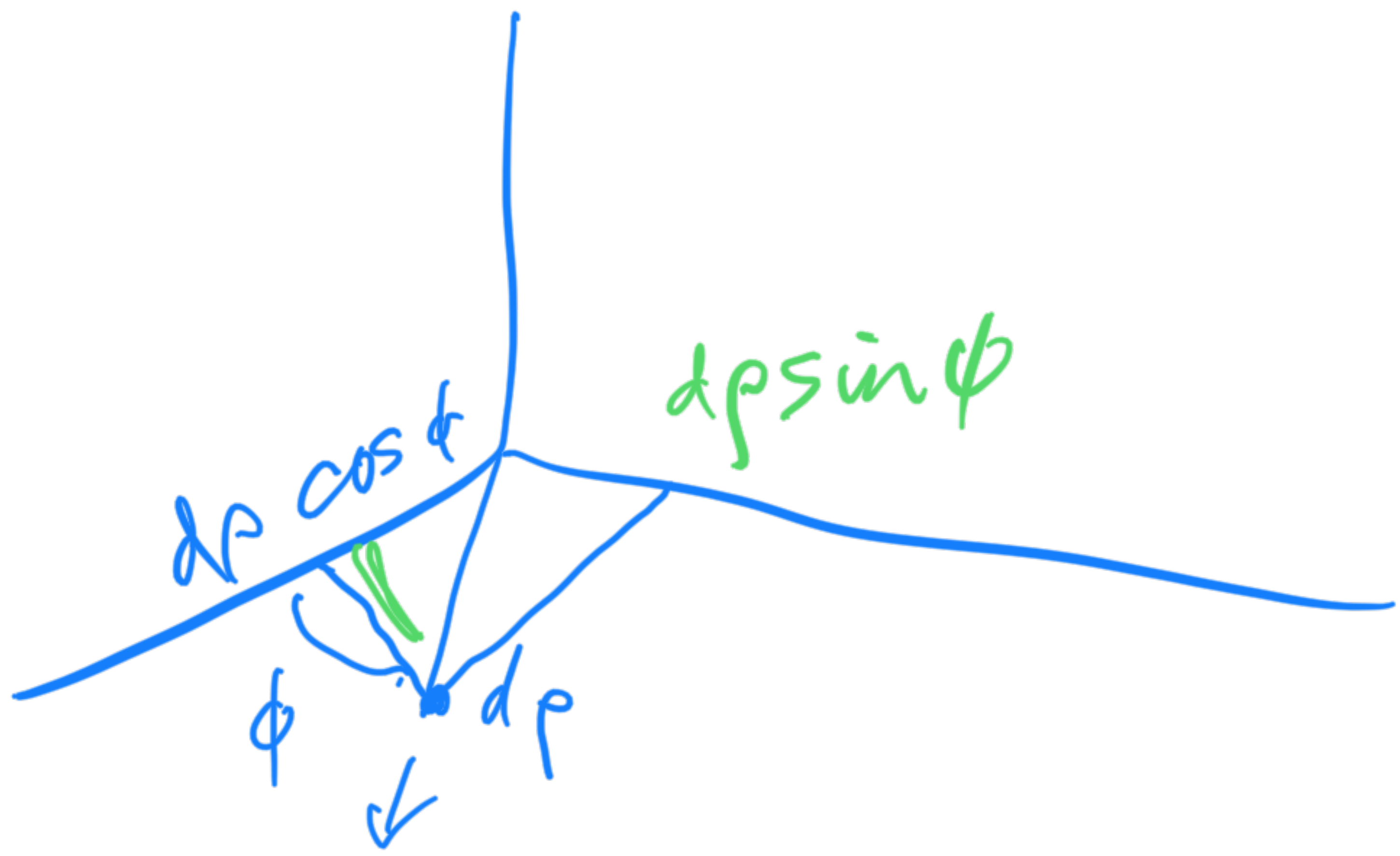
$$-\ell-1 = -2n-1$$

$$\ell = 2n$$

$$\delta(r-r') = \frac{2r^2}{a^3}\sum_{n=1}^{\infty}\frac{j_\ell(z_{\ell,n}r/a)\,j_\ell(z_{\ell,n}r'/a)}{j_{\ell+1}^2(z_{\ell,n})}$$

$$d\vec{p} = \hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz$$

$$e_\rho = \frac{\partial \vec{p}}{\partial \rho}$$



$$e_\phi = \frac{\partial \vec{p}}{\partial \phi}$$

$x = \rho\cos\phi \qquad y = \rho\sin\phi$

$dx = -\rho\sin\phi\, d\phi \qquad dy = \rho\cos\phi\, d\phi$

$d\vec{p} = -\hat{x}\sin\phi\, d\phi + \hat{y}\cos\phi\, d\phi$

$$e_\phi = \frac{\partial \vec{p}}{\partial \phi} = \hat{\phi} = \underline{-\hat{x}\sin\phi + \hat{y}\cos\phi}$$

$$e_z = \frac{\partial \vec{p}}{\partial z} = \underline{\hat{z}}$$

$$P_2(x) = \frac{3x^2-1}{2}$$

$$\sin^2\theta = 1 - \cos^2\theta$$
$$= 1 - x^2$$

$$x^i \longrightarrow x'^i$$

$$x'^i \longrightarrow x^i$$

$r^i = aV^i + bF^i$ no sum

+

$$G^k = V^{ki} B_i = \sum_i V^{ki} B_i$$

$$A'_{kl} C'^{k} = \frac{\partial x^j}{\partial x'^l} A_{kj} C^k$$

$$ds^2 = \eta_{ik}\, dx^i\, dx^k$$ invariant under Lorentz transformations

N

$$ds^2 = g_{ik}\, dx^i dx^k$$

invariant under general coordinate transformations

g_{ik} rank-2 tensor

$$g'_{ik} = \frac{\partial x^j}{\partial x'^i} \frac{\partial x^l}{\partial x'^k} g_{jl}$$

$$A = A_i\, dx^i$$ 1-form

$$ds = \frac{\partial s}{\partial x^i} dx^i$$

extension derivative

$$dA = \partial_j A_i\, dx^j \wedge dx^i$$

$$= -\partial_j A_i\, dx^i \wedge dx^j$$

$$= \partial_1 A_2\, dx^1 \wedge dx^2 + \dots$$

$$\partial_2 A_1\, dx^2 \wedge dx^1$$

$+ \partial_2 A_1 \, dx^2 \ldots$

$$= (\partial_1 A_2 - \partial_2 A_1)\, dx^1 \wedge dx^2 + \cdots$$

$$= \underline{F_{12}}\, dx^1 \wedge dx^2 + \cdots$$

$$ddA = d(\partial_j A_i \, dx^j \wedge dx^i)$$

$$= \partial_k \partial_j A_i \wedge dx^k \wedge dx^j \wedge dx^i$$

symmetric in j & k

antisymmetric j & k

So $ddA = 0.$

$$A^{i}_{,jk} = \frac{\partial}{\partial x^i} \frac{\partial^2 A}{\partial x^j \partial x^k}$$

$$\partial_i S = S_{,i} = \frac{\partial S}{\partial x^i}$$

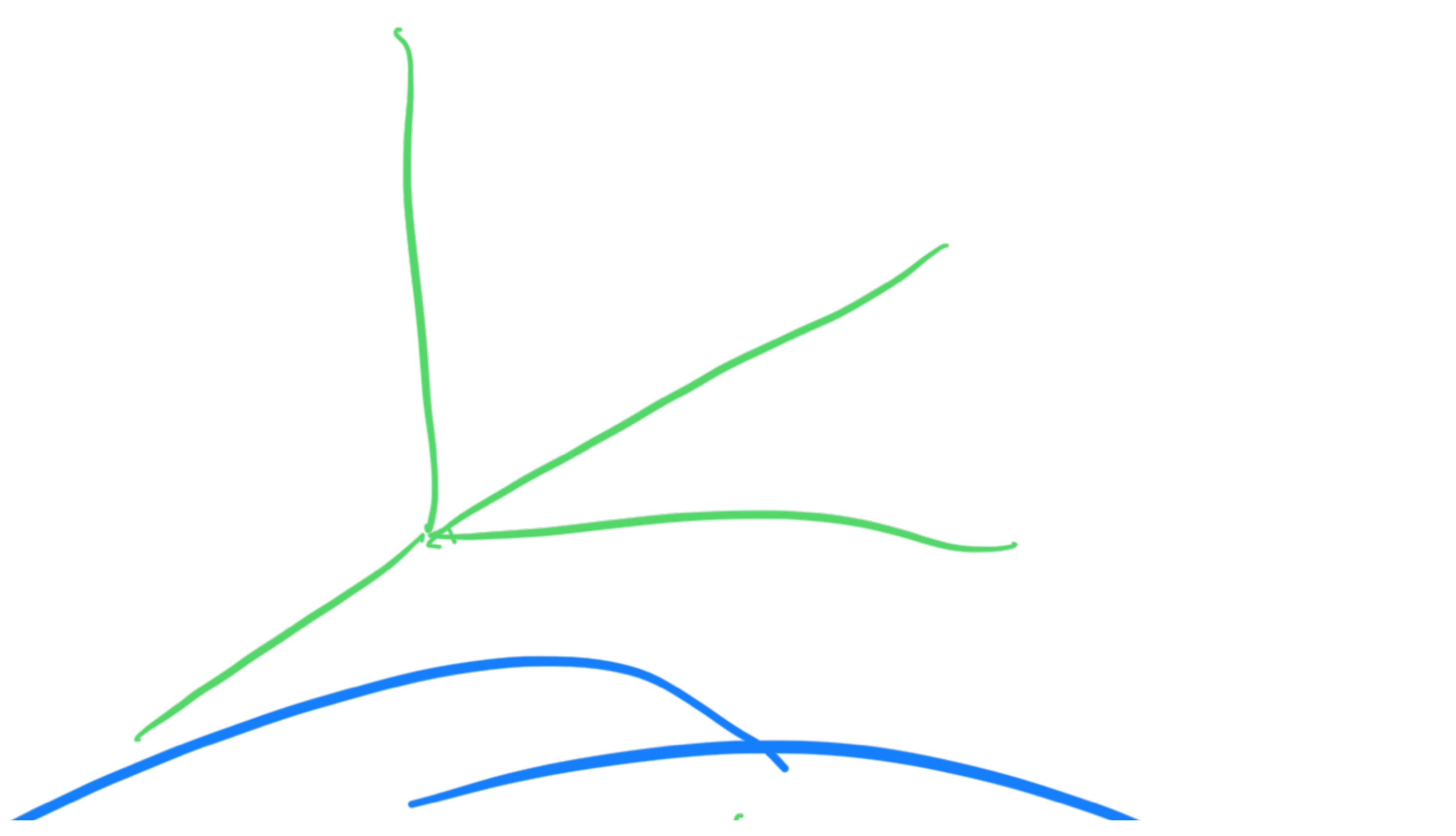
$$\partial^i s = s^{,i} = \frac{\partial}{\partial x_i}$$

$$D_i V^k = V^k{}_{;i}$$

$x'^i(x)$ smooth 1 to 1

$x^k(x')$

$$(T_{ij}{}^k)' = \sum_{l\,m\,n=0}^{3} \frac{\partial x^l}{\partial x'^i}\frac{\partial x^m}{\partial x'^j}\frac{\partial x'^k}{\partial x^n} T_{lm}{}^n$$



$$\eta = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} = \zeta$$

$$\zeta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\zeta \overset{\text{diag}}{=}_{n} (-1, -1, 1, 1, 1) \quad \text{open}$$

$$\zeta \overset{\text{diag}}{=}_{n} (-1, 1, 1, 1, 1) \quad \text{closed}$$

$$\zeta = \begin{pmatrix} -1 & & & & \\ & -1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}$$

$$\zeta = \begin{pmatrix} -1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}$$

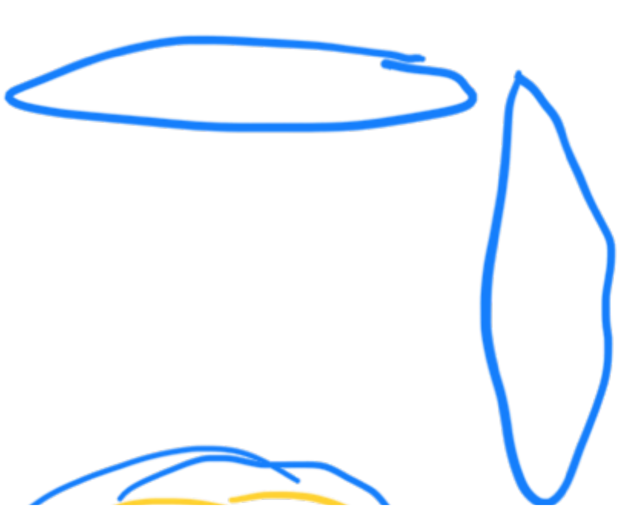
$$e_i = \frac{\partial p}{\partial x^i}$$

$$e_{i,l} = \frac{\partial e_i}{\partial x^l} = \frac{\partial^2 p}{\partial x^i \partial x^l}$$

$$= \frac{\partial^2 p}{\partial x^l \partial x^i} = e_{l,i}$$

$$\Gamma^k_{\ il} = \Gamma^k_{\ li}$$

$$e^k e_k = I$$



$e^k e_k \cdot e^l = e^k \delta^l_k = e^l$

$e_i \cdot e^k e_k = \delta^k_i \cdot e_k = e_i$

$e^k e_k = I$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\cancel{a A_k + b B^k}$

$ds^2 = \vec{dx}^2 - c^2 dt^2$

invariant under Lorentz transformations

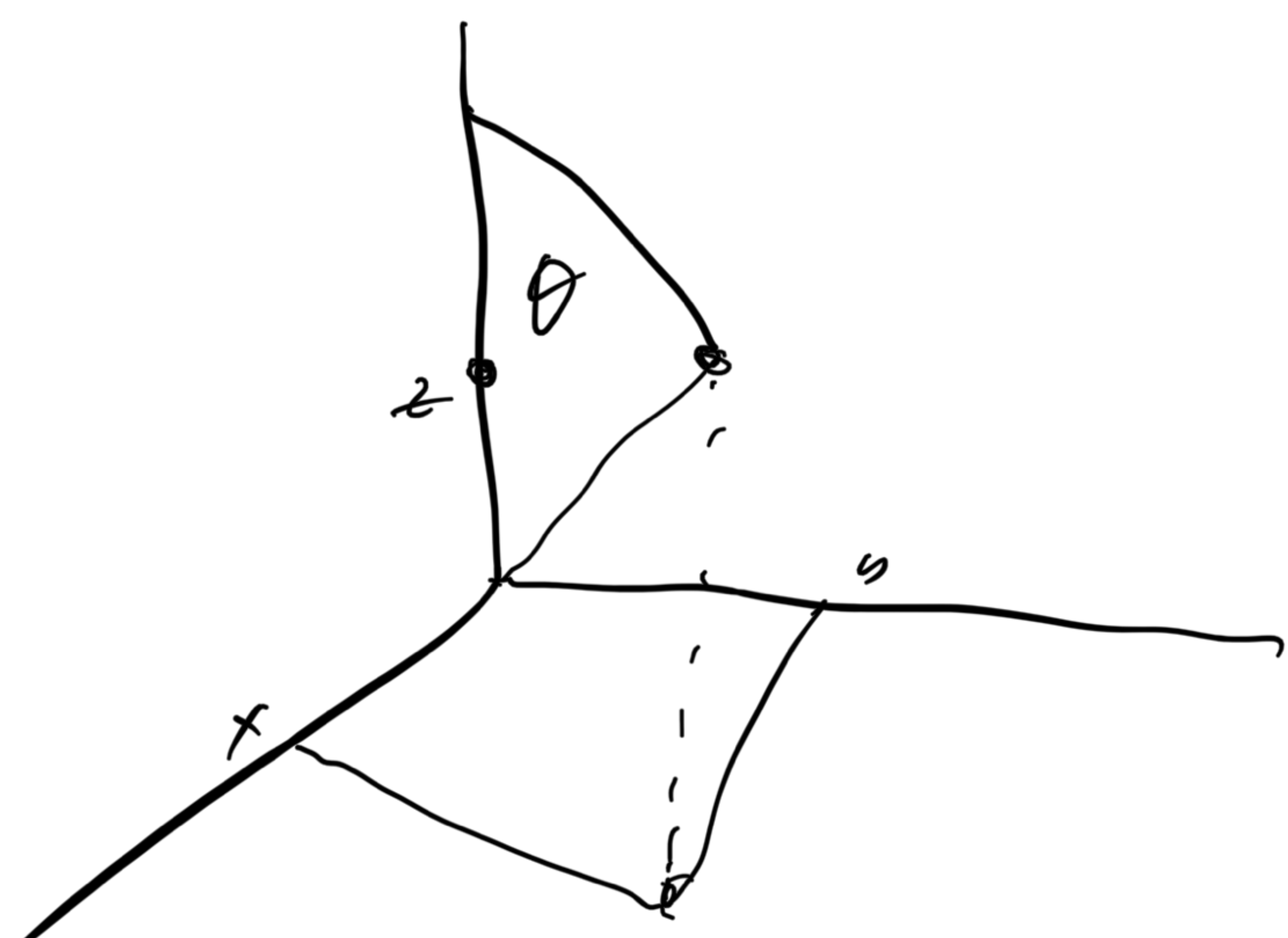
$ds^2 = g_{ik} dx^i dx^k$

inv. under general coordinate trans.

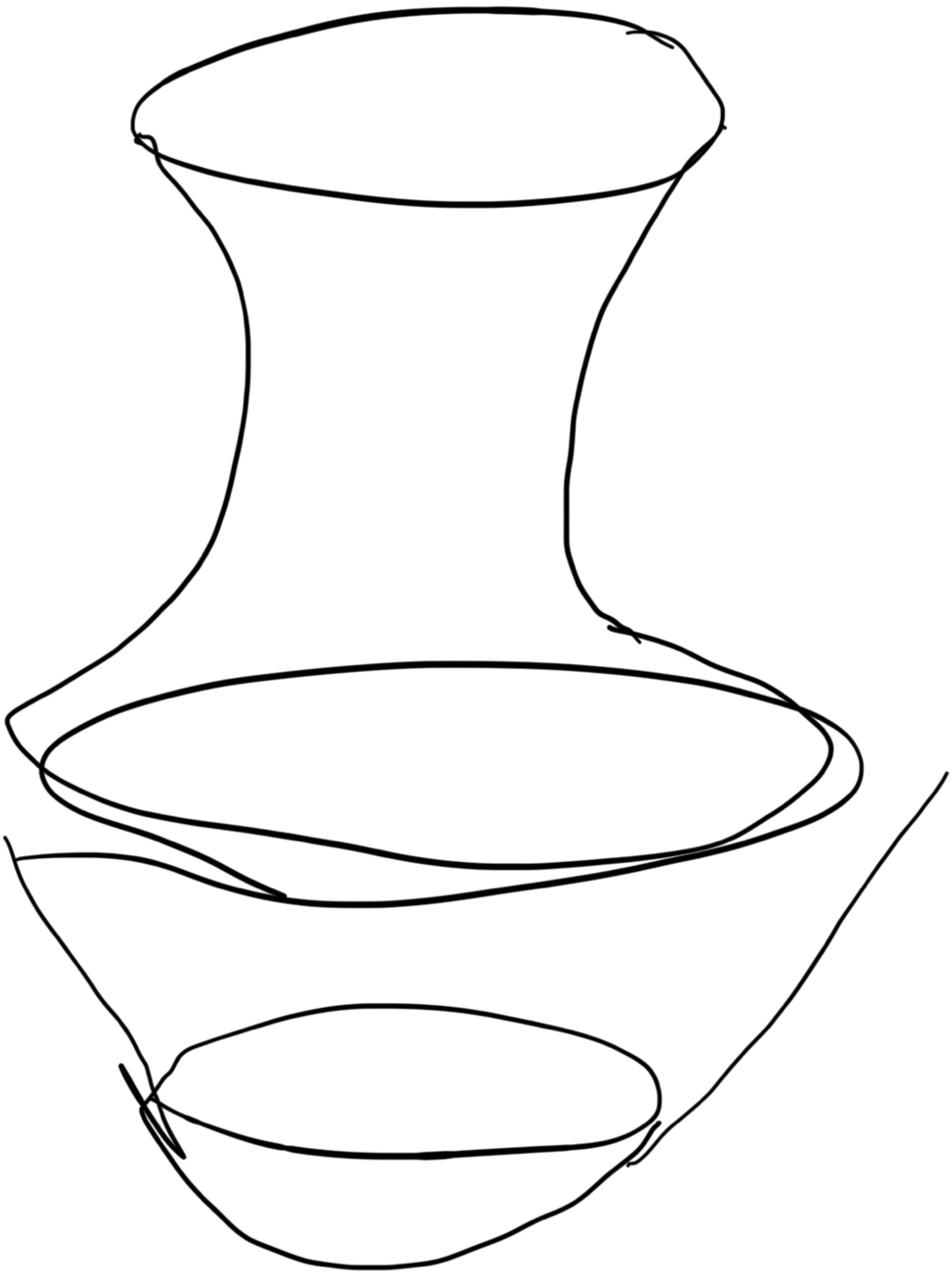
$$\eta = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & & & & & \\ 0 & 0 & -1 & & & & \\ 0 & & & 1 & & & \\ 0 & & & & 1 & & \\ & & & & & 1 & \\ & & & & & & 1 \end{pmatrix}$$

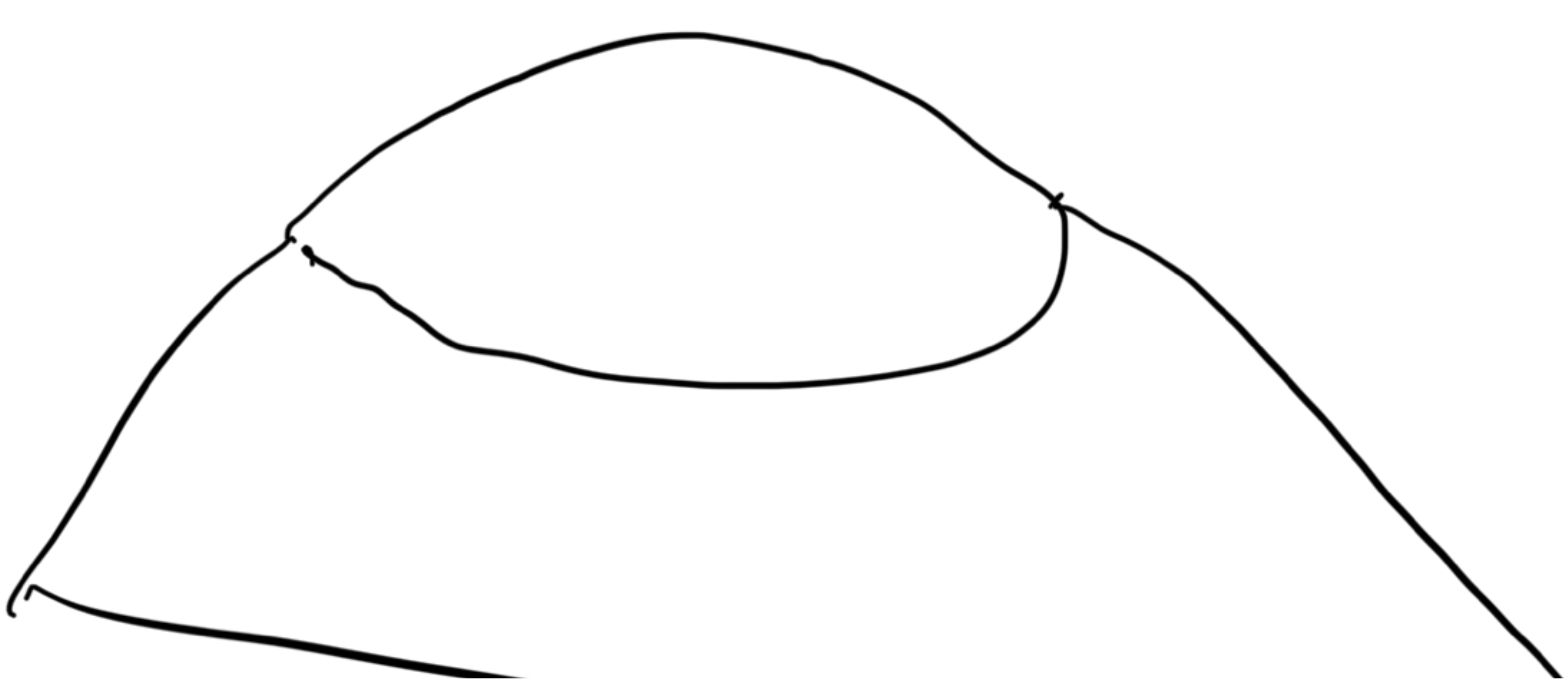
$$\eta = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

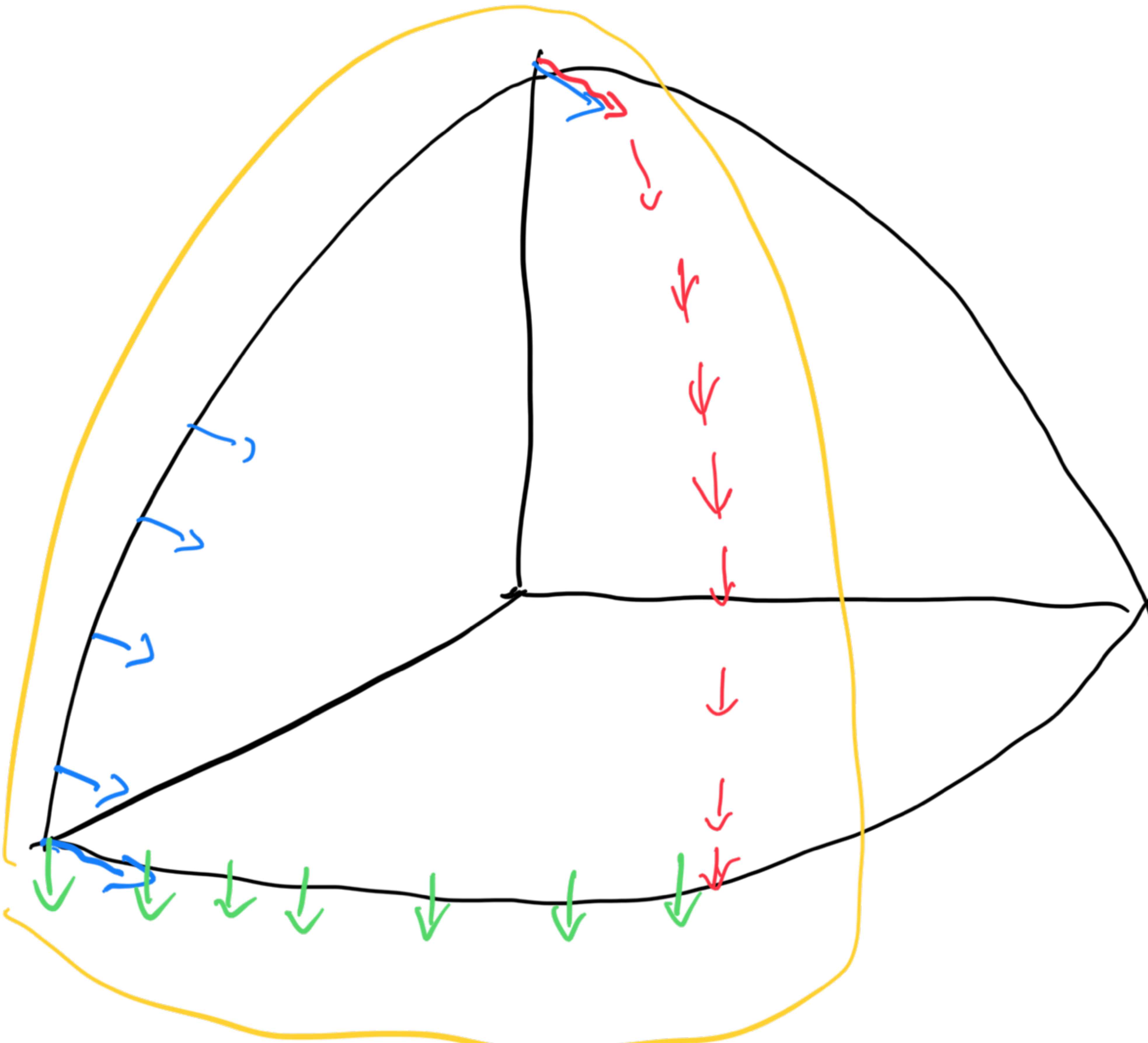
$$g = \begin{pmatrix} -c^2 & & & \\ & a & & \\ & & a & \\ & & & a \end{pmatrix}$$



$$\eta \overset{\text{diag}}{=} (1 \; 1 \; 1) = \begin{pmatrix} 1 & 0 & 0 \\ \end{pmatrix}$$







$\Gamma_0 + \Gamma_{,1} \frac{\Delta x^1}{2}$

$$\oint \Gamma^k{}_{il} V_k \, dx^l$$

$$= \oint \left(\Gamma^{k(0)}_{il} + \Gamma_{,m} (x-x_0)^m \right) \left(V_{k0} + \Gamma^{(0)} V^{(0)m}_{,(x-x_0)} \right) dx^l$$

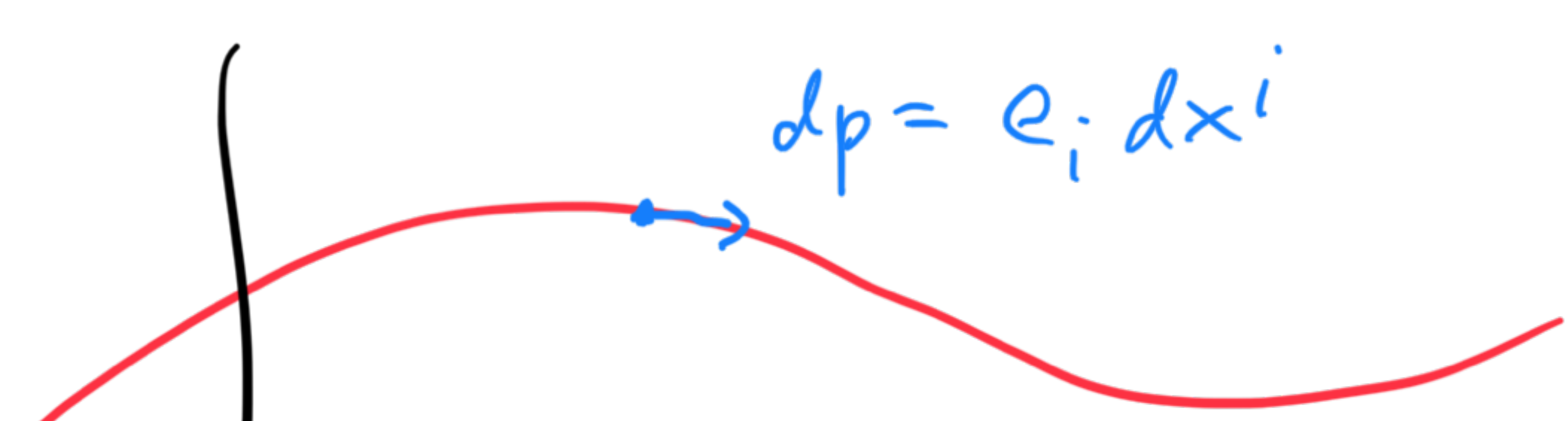
$$= \cancel{\Gamma^0 V_{k0} \oint dx^l} \to 0$$

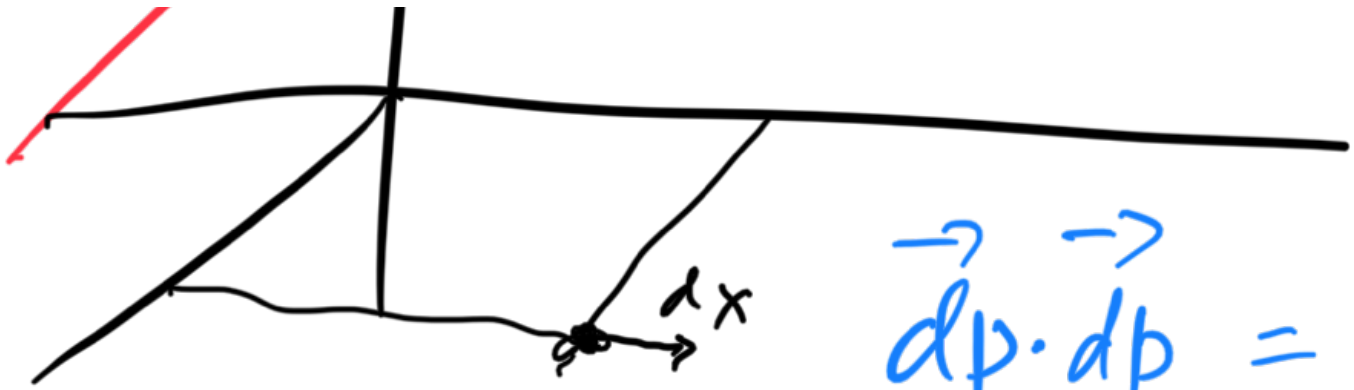
$$+ \int \Gamma_{,m} (x-x_0)^m V_{k0} \, dx^l$$

$$+ \int \Gamma^k \Gamma V (x-x_0) \, dx^l$$

$$[A, B] = AB - BA$$

$$\{A, B\} = AB + BA$$





$$\vec{dp}\cdot\vec{dp} = g_{ik}\,dx^i dx^k$$

$$= e_i\cdot e_k\,dx^i dx^k$$

$$e_1 = \begin{pmatrix} a \\ b \end{pmatrix} \qquad e_2 = \begin{pmatrix} c \\ d \end{pmatrix}$$

$$e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$e_1 e^1 = e_1 \otimes e^1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$e_2 e^2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$e_k e^k = e_k \otimes e^k = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

$$g_{ni,l} = (e_n\cdot e_i)_{,l}$$

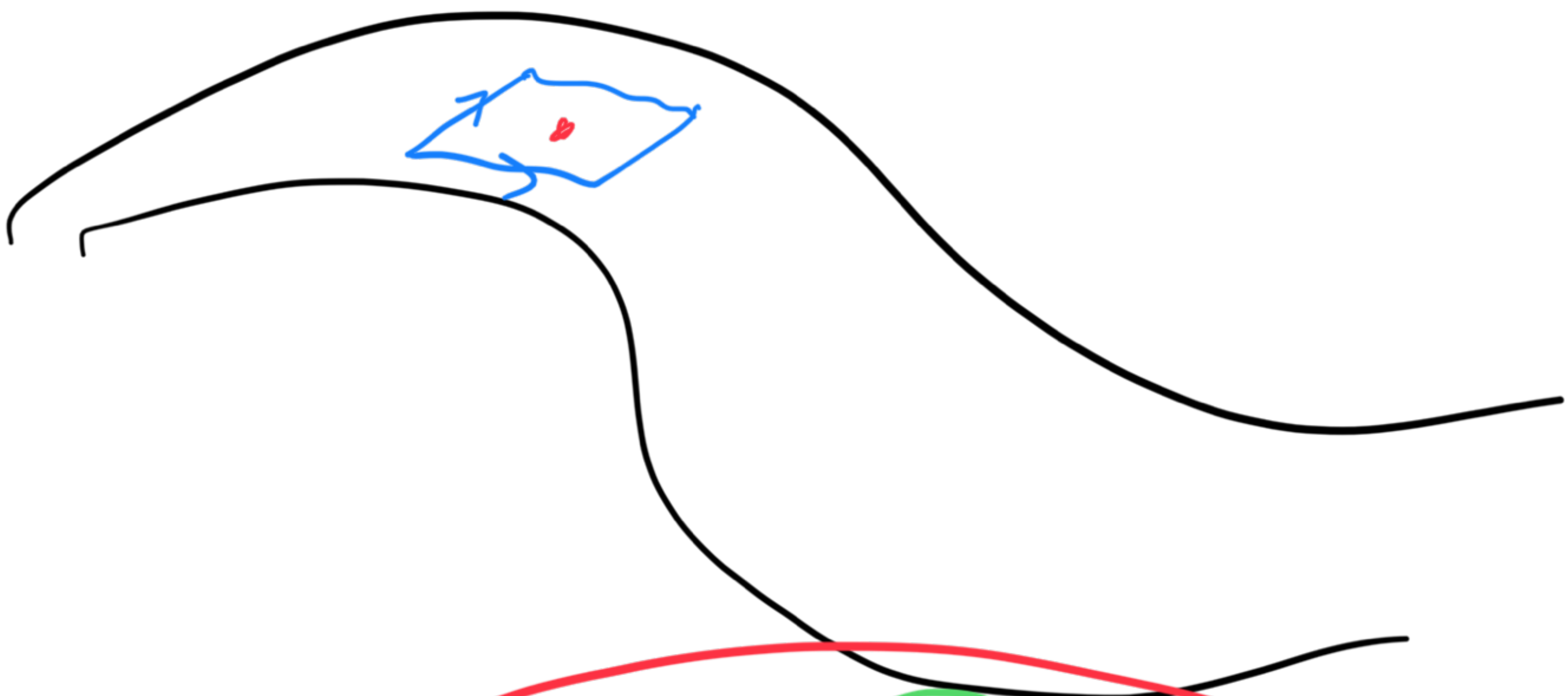
$$= e_{n,l}\cdot e_i + e_n\cdot e_{i,l}$$

$$\Gamma_{nil} = e_n\cdot e_{i,l}$$

km

$$\Gamma^{k}_{i\ell} = g^{km} e_m \cdot e_{i,\ell}$$

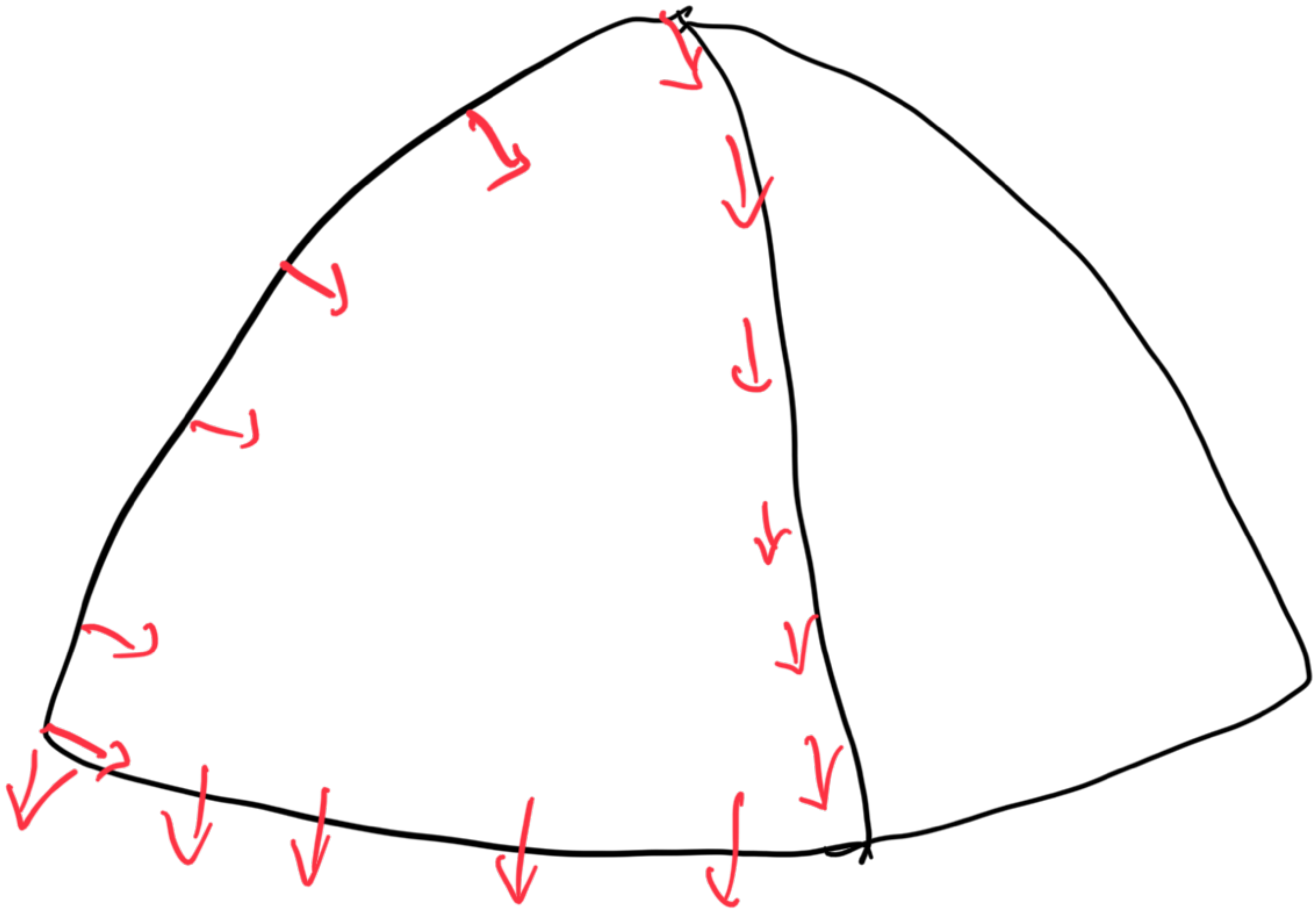
$$= e^{k} \cdot e_{i,\ell}$$

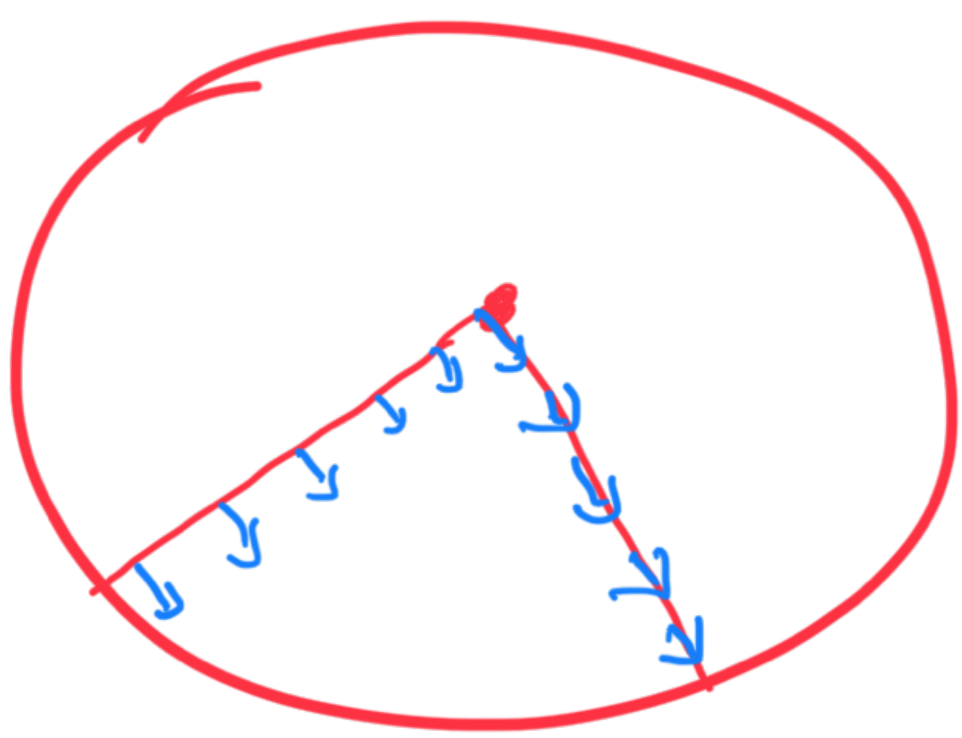


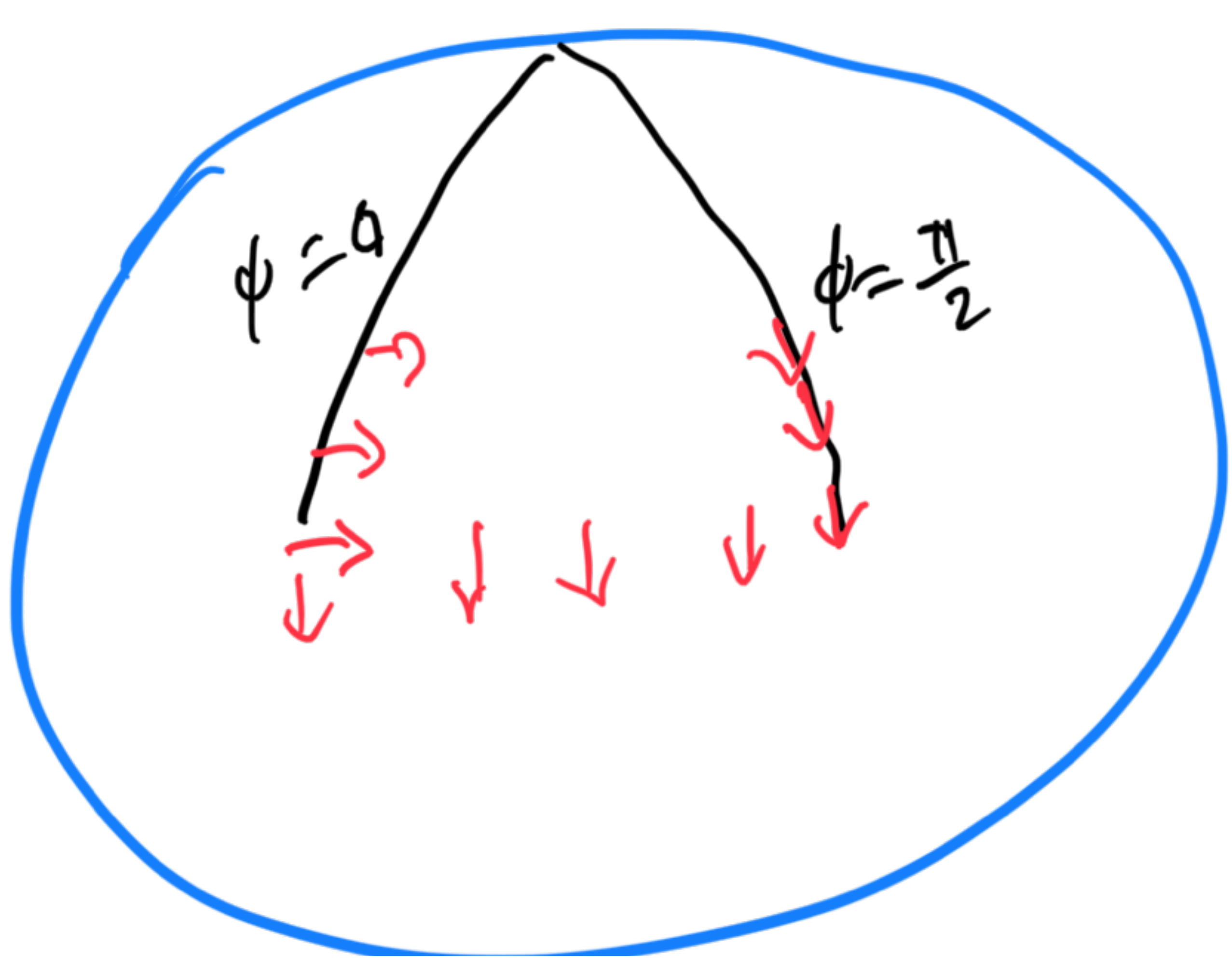
$$V^{i}_{,\ell} \frac{dx^{\ell}}{du} = -\Gamma^{i}_{k\ell} \frac{dx^{\ell}}{du}$$

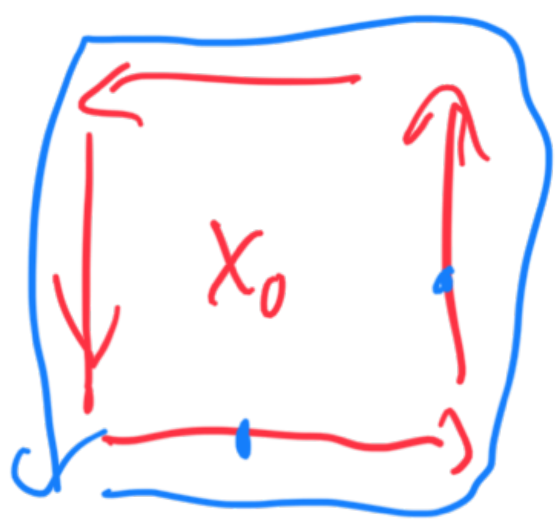
$$\Rightarrow \left(V^{i}_{,\ell} + \Gamma^{i}_{k\ell}\right) \frac{dx^{\ell}}{du} = 0$$

$$D_{\ell} V^{i} \frac{dx^{\ell}}{du} = 0$$





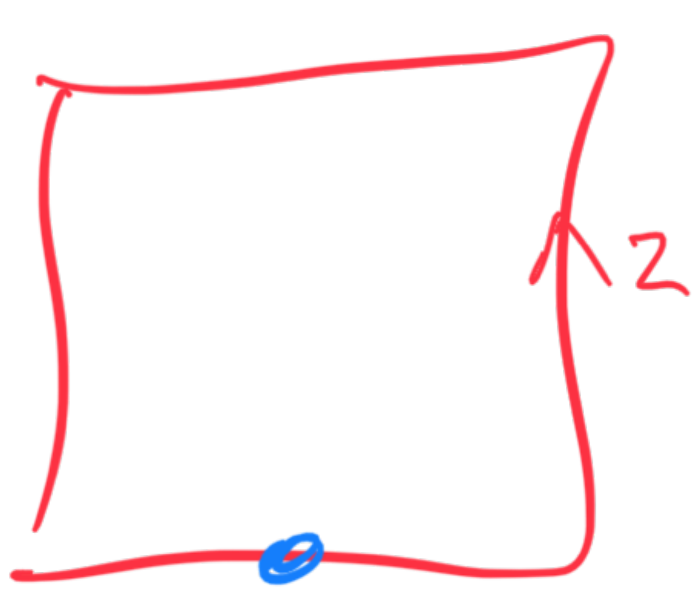

ϕ=0
ϕ=π/2



$$\oint (x-x_0)^n dx^l = -x_0^n \int dx^l$$

$$+ \oint x^n dx^l =$$

$$\int (x^1-x_0^1) dx^2 = (x^1-x_0^1) a = -\frac{a}{2} a$$



$$\int_0^a \left(x-\frac{a}{2}\right) dx = \frac{a^2}{2} - \frac{a^2}{2} = 0$$

$$S_G = \cancel{\frac{8\pi G}{3}} \int R \sqrt{g}\, d^4x$$

scalar

$$g = |\det(g_{ik})|$$

$$S_G = \frac{c^3}{16\pi G} \int R \sqrt{g}\, d^4x$$

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik}$$

energy-momentum tensor
of the matter fields.