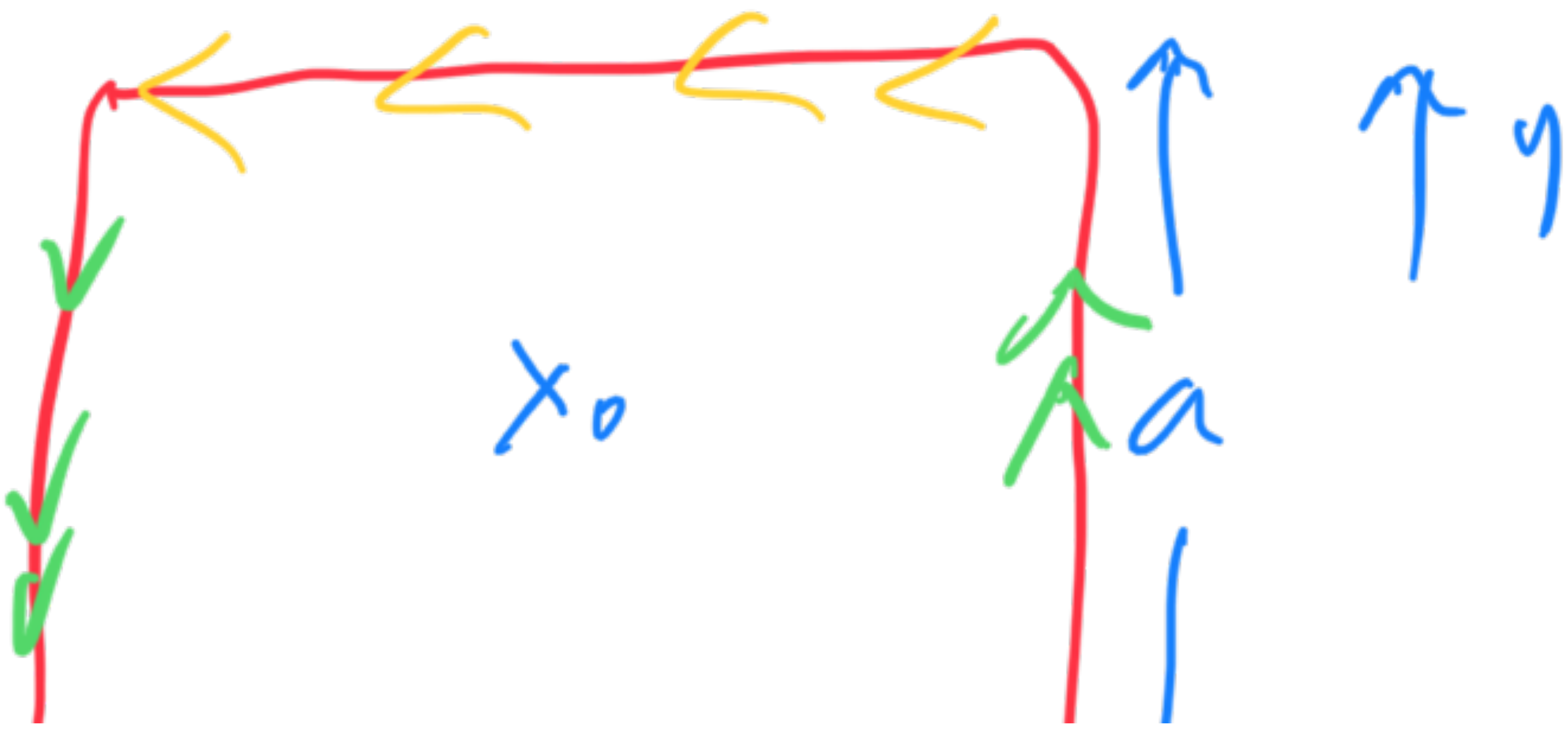
$$\oint \partial x^{\ell} = 0$$

$$\oint (x - x_0)^n \, dx^{\ell} = \pm a^2$$



$\to x$

$x^l = y$

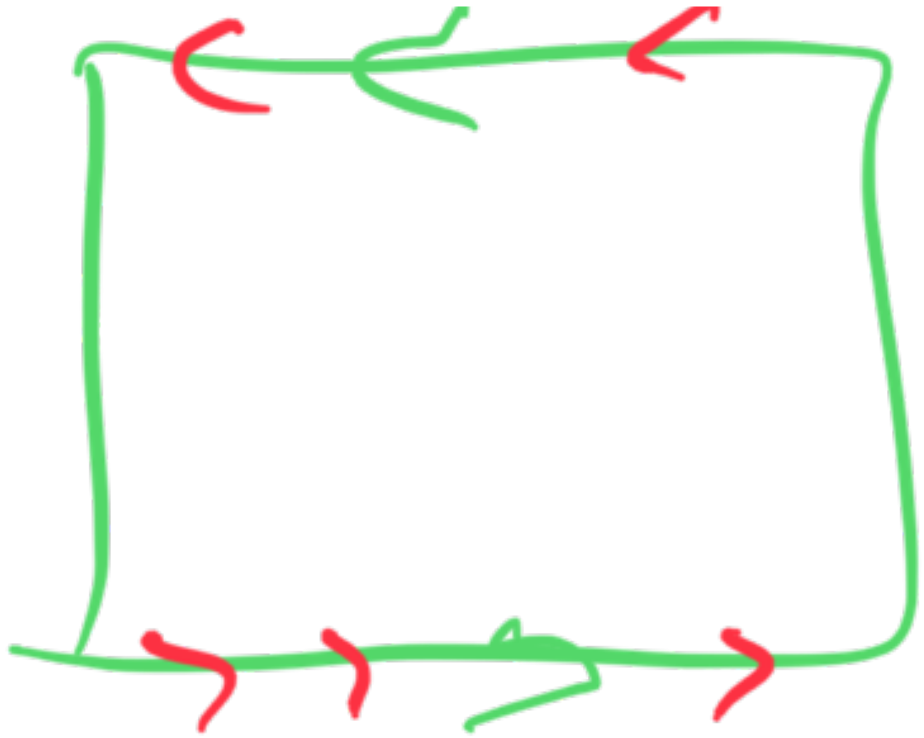
$x^n = x$

$$\oint \left(x - \frac{a}{2}\right) dy$$

$$= \left(a - \frac{a}{2}\right) a - a\left(-\frac{a}{2}\right)$$

$$= a^2 - \frac{a^2}{2} + \frac{a^2}{2} = a^2$$

$$\oint \left(x - \frac{a}{2}\right) dx = \int_0^a \left(x - \frac{a}{2}\right) dx + \int_a^0 \left(x - \frac{a}{2}\right) dx$$

 $= 0$

$$\oint \left(y - \frac{a}{2}\right) dx = \int_0^a \left(-\frac{a}{2}\right) dx + \int_a^0 \left(a - \frac{a}{2}\right) dy$$

$$= -\frac{a^2}{2} - \int_0^a \frac{a}{2}\, dx = -\frac{a^2}{2} - \frac{a^2}{2} = -a^2$$

$$d^4x \sqrt{|\det(g_{ik})|} \qquad d^4y \sqrt{|\det g_{ik}|}$$

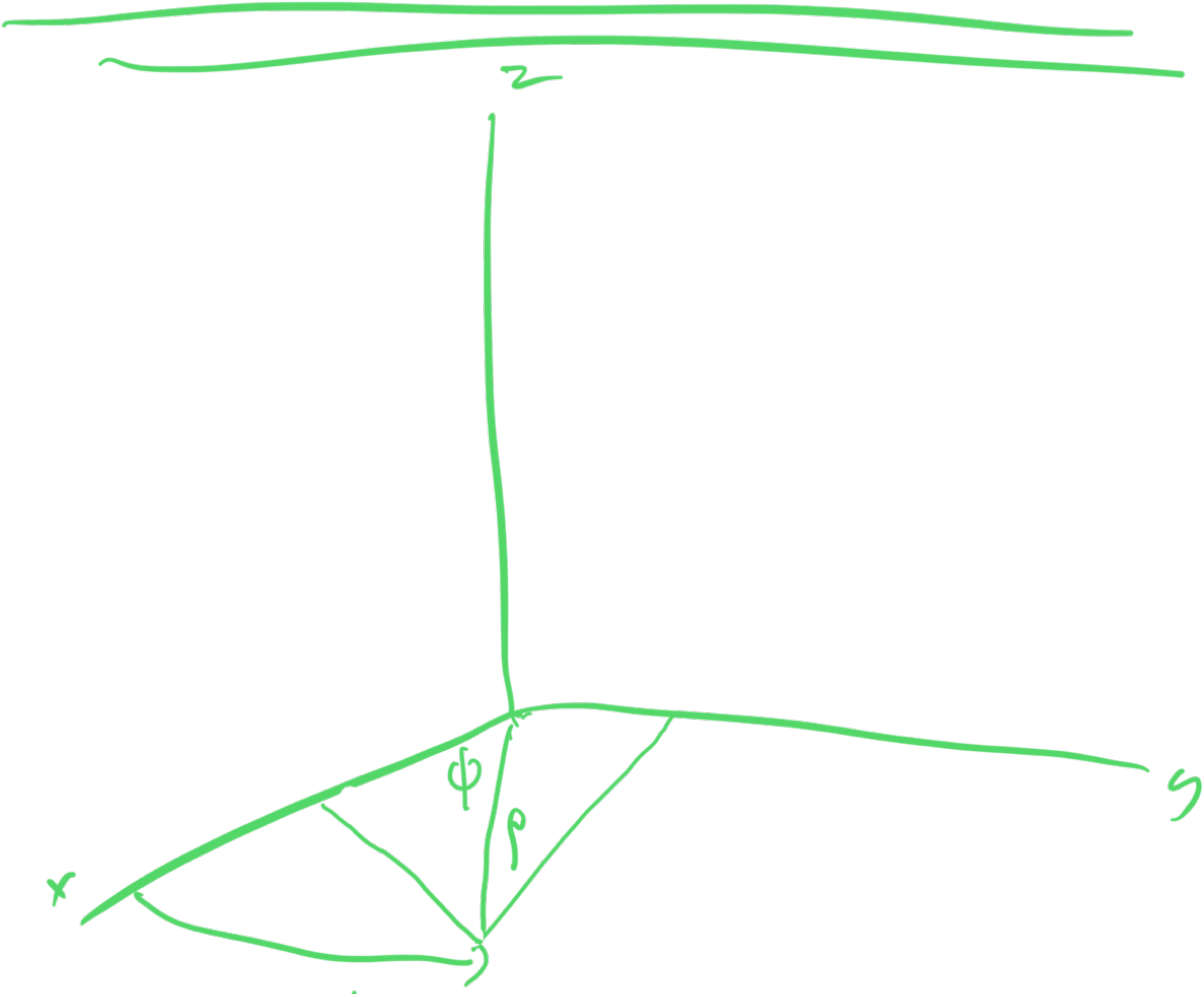
$$d^4x \, \det\left(\frac{\partial y}{\partial x}\right) \sqrt{\left| \frac{\partial x^{j}}{\partial y^{i}} \frac{\partial x^{\ell}}{\partial y^{k}} g_{j\ell}(x) \right|}$$

1 2 3

$$= d^4x \, \det\frac{\partial y}{\partial x} \det\frac{\partial x}{\partial y} \sqrt{|\det g(x)|}$$

$$= d^4x \sqrt{|\det g(x)|} = d^4y \sqrt{|\det g(y)|}$$

$$\sqrt{g} \, d^4x$$


z
φ
ρ
x
y

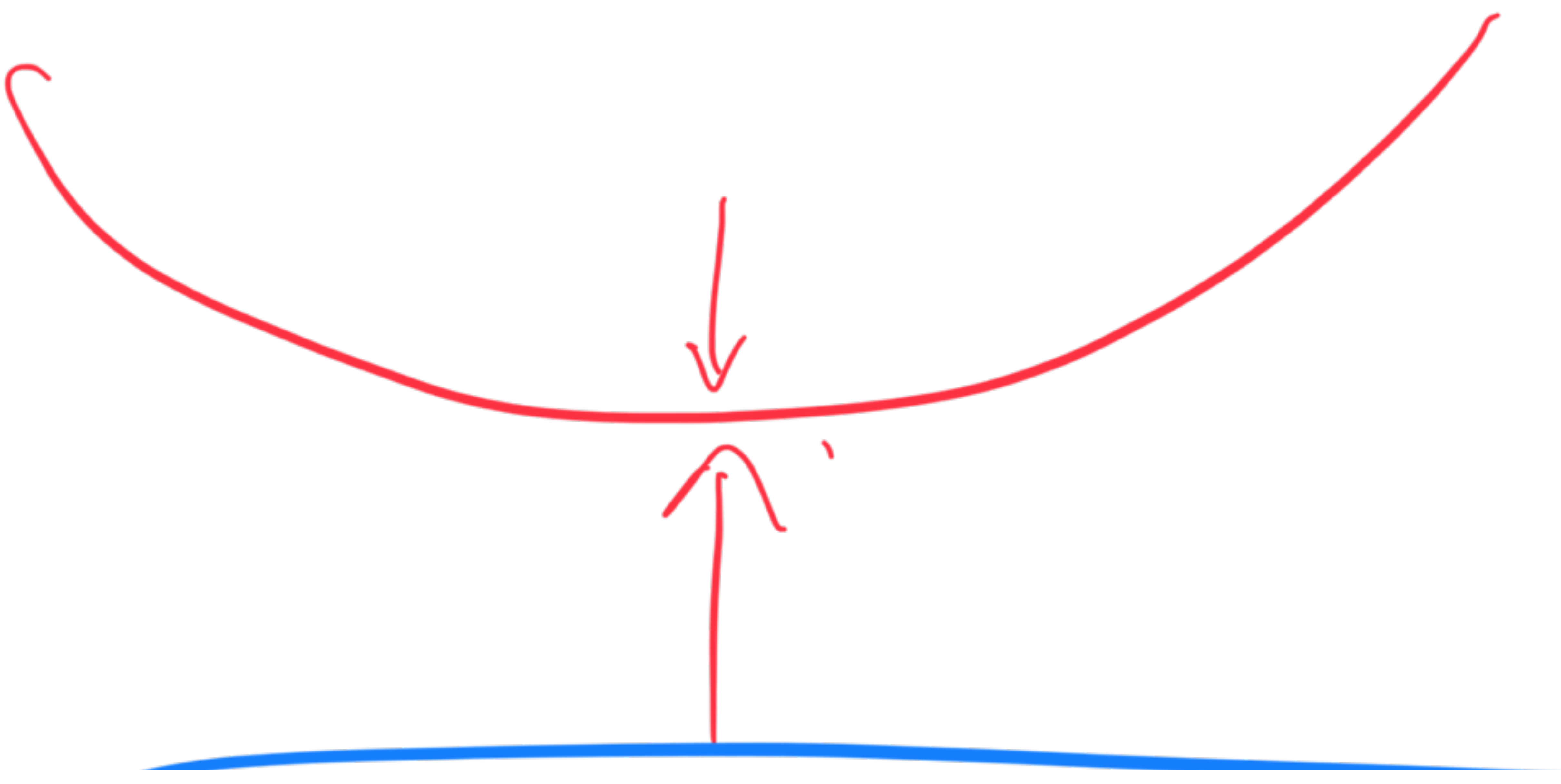
ϕ

$$dp^2 = e_t \cdot e_t \, dt^2 + e_\chi \cdot e_\chi \, d\chi^2 + e_\theta \cdot e_\theta \, d\theta^2 + e_\phi \cdot e_\phi \, d\phi^2$$

$$= -c^2 dt^2 + a^2 L^2 (\mathrm{ch}^2\chi - \mathrm{sh}^2\chi)\, d\chi^2 + a^2 L^2 \mathrm{sh}^2\chi \, d\theta^2 + a^2 L^2 \mathrm{sh}^2\chi \sin^2\theta \, d\phi^2$$

$$\lambda = \frac{n}{p}$$

$$n = (-1\ ,\ ,\ ,\ )$$



$$S = S_G + S_m$$

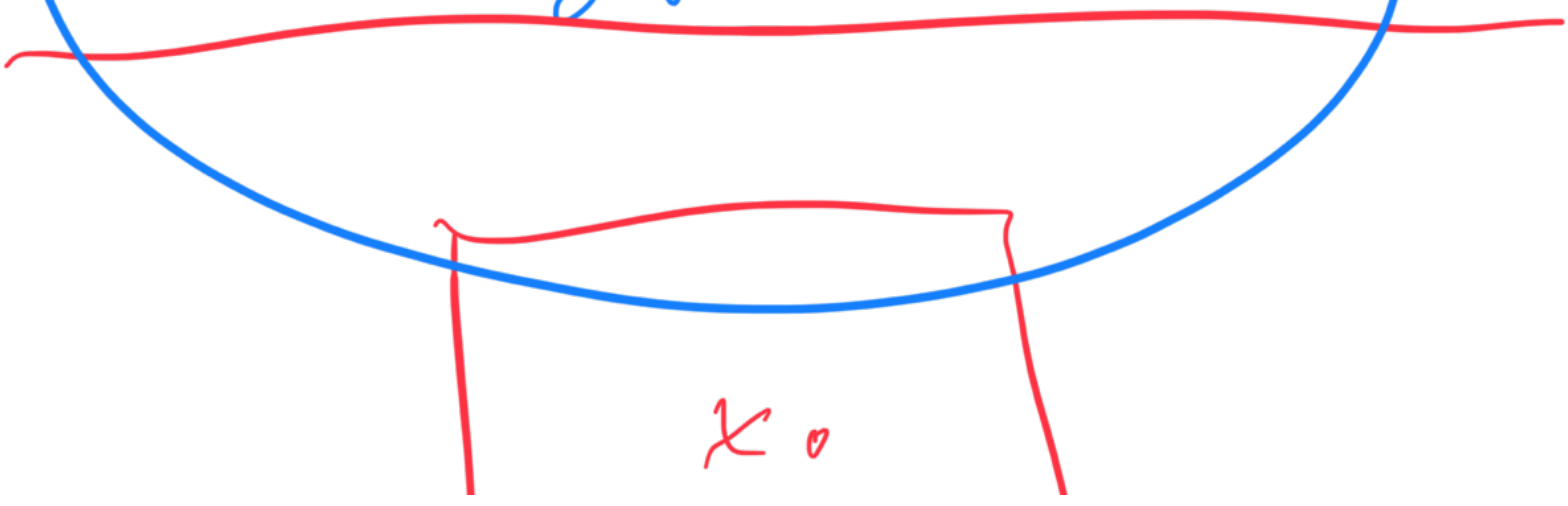
$$g^{li} R_{ik} - \frac{1}{2} g^{li} g_{ik} R = \frac{8\pi G}{c^4} g^{li} T_{ik}$$

$$R^{l}{}_{k} - \frac{1}{2} \delta^{l}_{k} R = \frac{8\pi G}{c^4} T^{l}{}_{k}$$

$$R - \frac{4}{2} R = \frac{8\pi G}{c^4} T$$

$$R = -\frac{8\pi G}{c^4} T$$

$$\phi \sim -\frac{\partial u}{\partial V} < 0$$



$$\oint dx^l = 0$$

$$u^i = \frac{dx^i}{d\tau}$$

$$\delta\sqrt{g} = \delta\sqrt{|\det(g_{ik})|}$$

$$= \frac{1}{2} \frac{\delta \det g}{\sqrt{g}}$$

$$= \frac{1}{2} \frac{\det g}{g\sqrt{g}} \delta \det g$$

$$\delta^i_l = g^{ik} g_{kl}$$

$$\lambda = \eta = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$dt = \frac{d\tau_\infty}{\sqrt{\left(1 - \frac{2GM}{c^2 r}\right)}} \rightarrow \infty$$

$$g_{00,i} = 0$$

$$g_{rr,0} = \frac{2a\dot{a}}{\left(1 - kr^2/L^2\right)}$$

$$g_{\theta\theta,0} = 2a\dot{a}r^2$$

$$g_{\phi\phi,0} = 2a\dot{a}r^2\sin^2\theta$$

$$\Gamma_{i\theta\theta} = \frac{1}{2}\left(2g_{i\theta,\theta} - g_{\theta\theta,i}\right)$$

$$\Gamma_{t\theta\theta} = \frac{1}{2}\left(2\underbrace{g_{t\theta,\theta}}_{=0} - g_{\theta\theta,t}\right)$$

$$\Gamma_{kil} = \frac{1}{2}\left(g_{ki,l} + g_{kl,i} - g_{li,k}\right)$$

$$\Gamma \ldots = \frac{1}{2}\left(\ldots \right) = -a^2 r$$

$$\Gamma^r{}_{\theta\theta} = \frac{1}{2}\left(-g_{\theta\theta,r}\right)$$

$$H_0 = h\, 100 \text{ km/s/Mpc}$$

$$h \sim 0.7$$

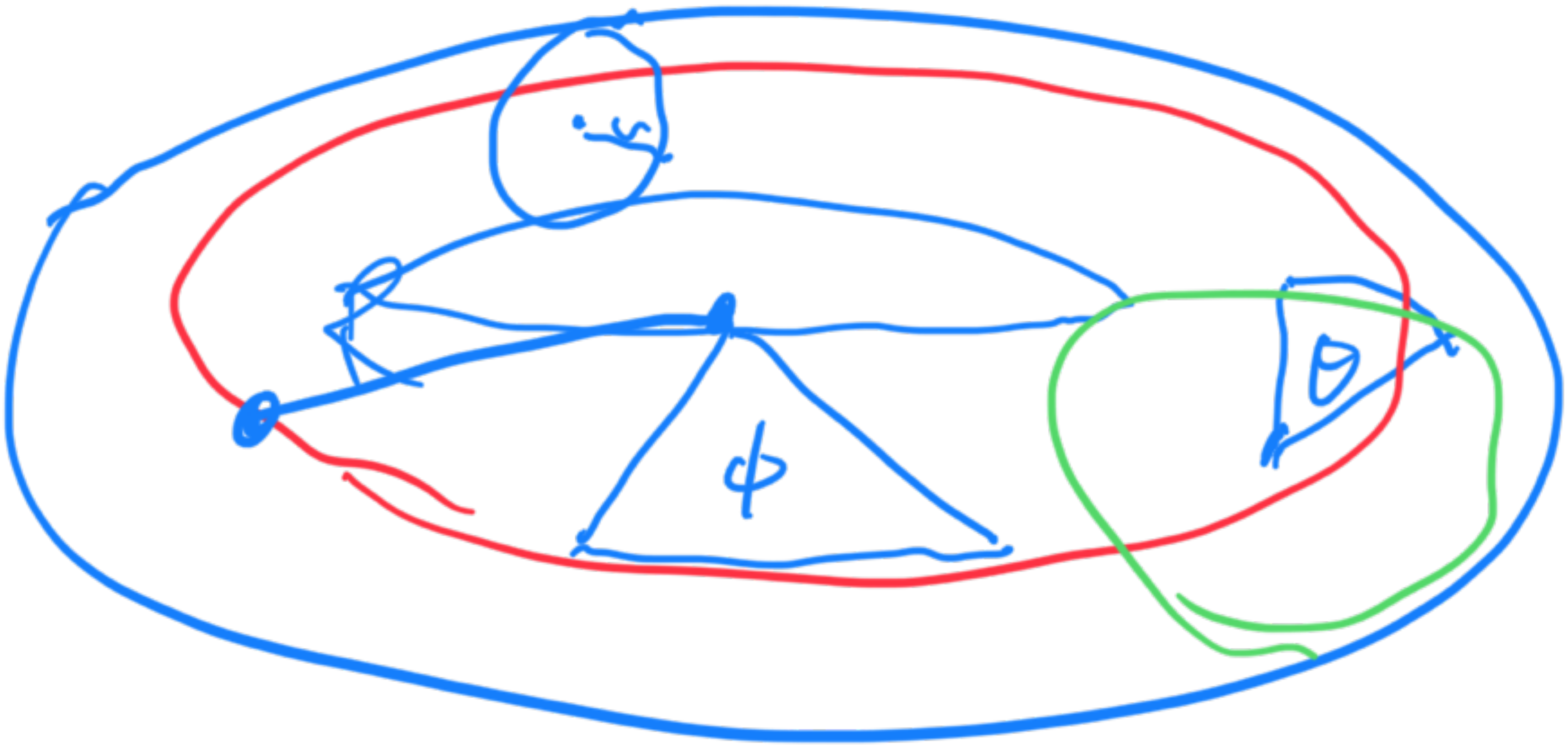
$$\frac{\dot{a}^2}{a^2} = \Omega_\Lambda + \frac{\Omega_k}{a^2} + \frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4}$$

$$\dot{a}^2 = \Omega_\Lambda a^2 + \Omega_k + \frac{\Omega_m}{a} + \frac{\Omega_r}{a^2}$$

$$\dot{a} = \frac{da}{dt} = \frac{da}{aH} = \frac{da}{a\left(\frac{da}{dt}\right)}$$

$$= \frac{1}{H}\frac{da}{a} = \frac{1}{H_0}\frac{H_0}{H}\frac{da}{a}$$

$$z = \frac{1}{a} - 1 = \frac{1}{a} - \frac{a}{a} = \frac{1-a}{a}$$



$$\frac{\partial}{\partial x^n}\left(\frac{\partial x^m}{\partial x'^i} e_m\right)$$

$$\partial^2 x^m$$

$$= \frac{\partial e_m}{\partial x^n} \frac{\partial \lambda}{\partial x'^i} + e_m \frac{\quad}{\partial x^n \partial x'^i}$$

$$A'_m = A_m + \partial_m \lambda$$

$$\frac{\partial x'^i}{\partial x^j} \qquad GL(4,R)$$

$$D^\dagger(g) D(g) = I$$

$$U(a|1\rangle + b|2\rangle)$$

$$= aU|1\rangle + bU|2\rangle$$

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}$$

$$\ell(x) = \begin{pmatrix} \nu(x) \\ e(x) \end{pmatrix}$$

$$\psi^{\dagger\prime} = \psi^{\dagger} U^{\dagger}$$

$$\psi' = U\psi$$

$$(\psi^\dagger \psi)' = \psi^\dagger U^\dagger U \psi = \psi^\dagger \psi$$

$$(\partial_i U)\psi + U\partial_i\psi + A_i' U\psi$$

$$= U\partial_i\psi + U A_i \psi$$

$$\partial_i U + A_i' U = U A_i$$

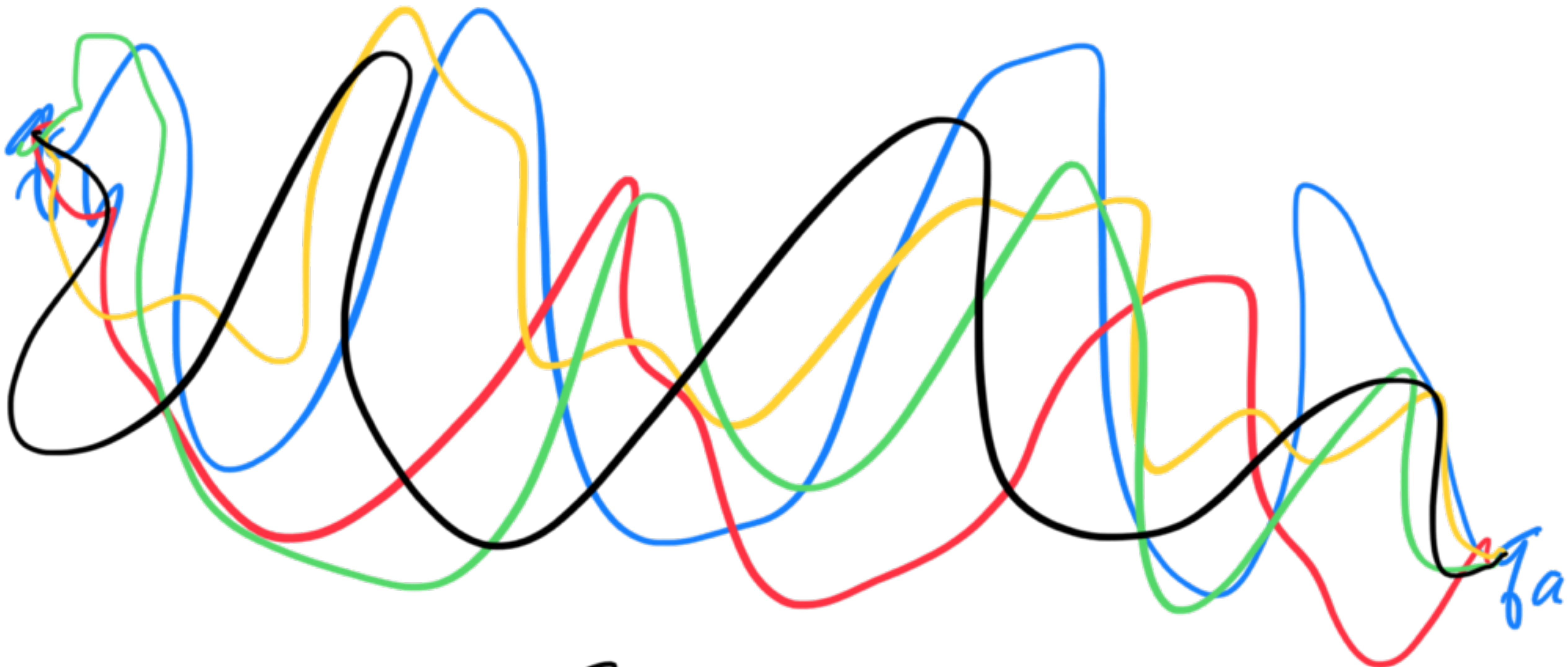
$$A'_i U = U A_i - \partial_i U$$

$$\boxed{\underset{\sim}{A'_i} = \underset{\sim}{U}\,\underset{\sim}{A_i}\,\underset{\sim}{U}^{-1} - (\partial_i \underset{\sim}{U})\underset{\sim}{U}^{-1}}$$

Path Integrals

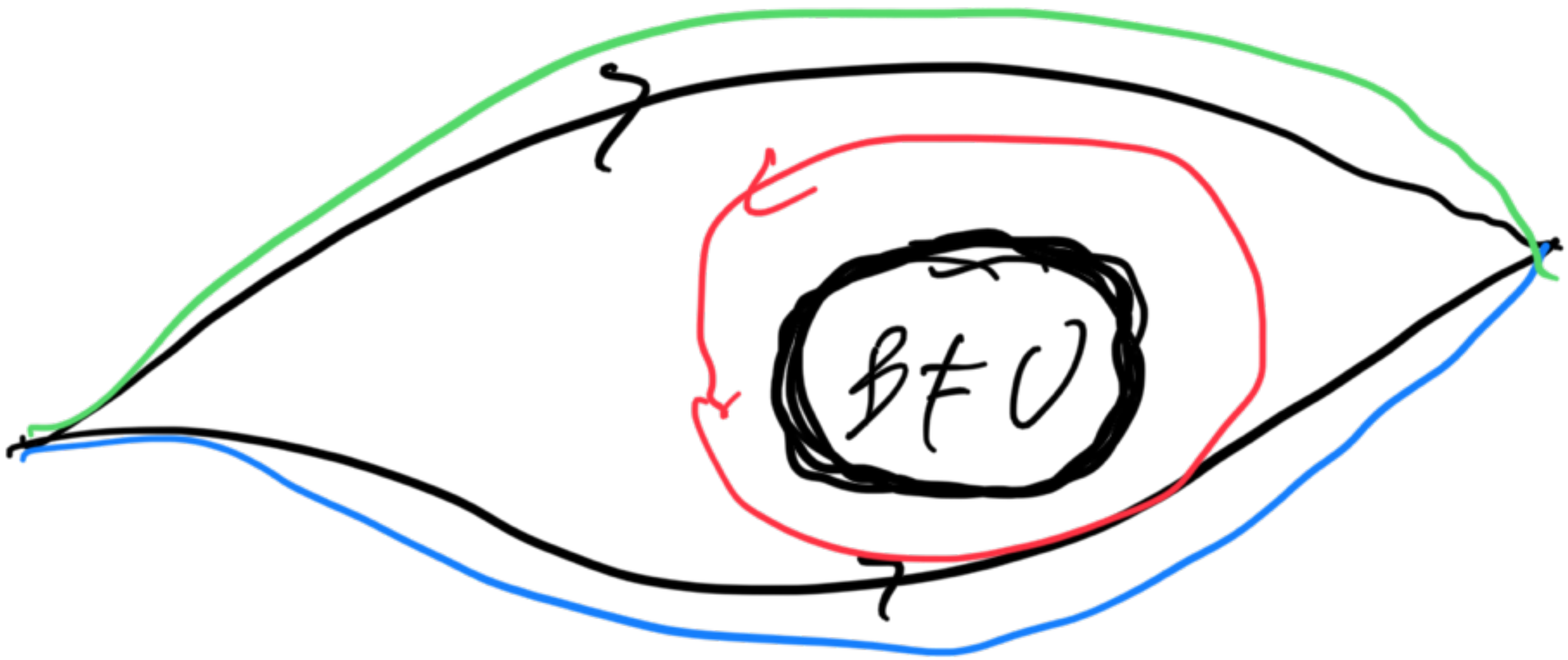
$$\beta = \beta = \frac{1}{kT}$$

$$[q, p] = qp - pq = i\hbar$$



$$\langle q_b | e^{-i\frac{T}{\hbar}\frac{p^2}{2m}} | q_a \rangle$$

$$= \int dp\, e^{ip(q_b - q_a) - i\frac{p^2 T}{\hbar 2m}}$$



$$A_i j^i \to A_i \dot{q}^i$$

$$\int A \cdot dq = \int \nabla \times A \cdot d\sigma = \int B \cdot d\sigma$$

$$= \Phi \qquad \hbar$$

$$\beta = \frac{1}{kT} \qquad \epsilon = \frac{\beta}{n}$$

$$\mathrm{Tr}\left(|q_0\rangle R \langle q_a|\right) = \int \langle q_a | R | q_a \rangle \, dq_a$$

$$\phi(\vec{x})|\phi'\rangle = \phi'(\vec{x})|\phi'\rangle$$

$$\pi(\vec{x})|\pi'\rangle = \pi'(\vec{x})|\pi'\rangle$$

$$\langle q_1 | p' \rangle = e^{i q_1 p'/\hbar} \qquad e^{i q_1 p'/\hbar}$$

$$\frac{\cdots}{\sqrt{2\pi\hbar}} = \frac{\cdots}{\sqrt{h}}$$

$$\langle p' | q_a \rangle = \frac{e^{-i q_a p'/\hbar}}{\sqrt{h}}$$

$$\langle q_1 | p' \rangle \langle p' | q_a \rangle = \frac{e^{i p'(q_1 - q_a)/\hbar}}{2\pi\hbar}$$

$$S = \int K - V \, dt$$

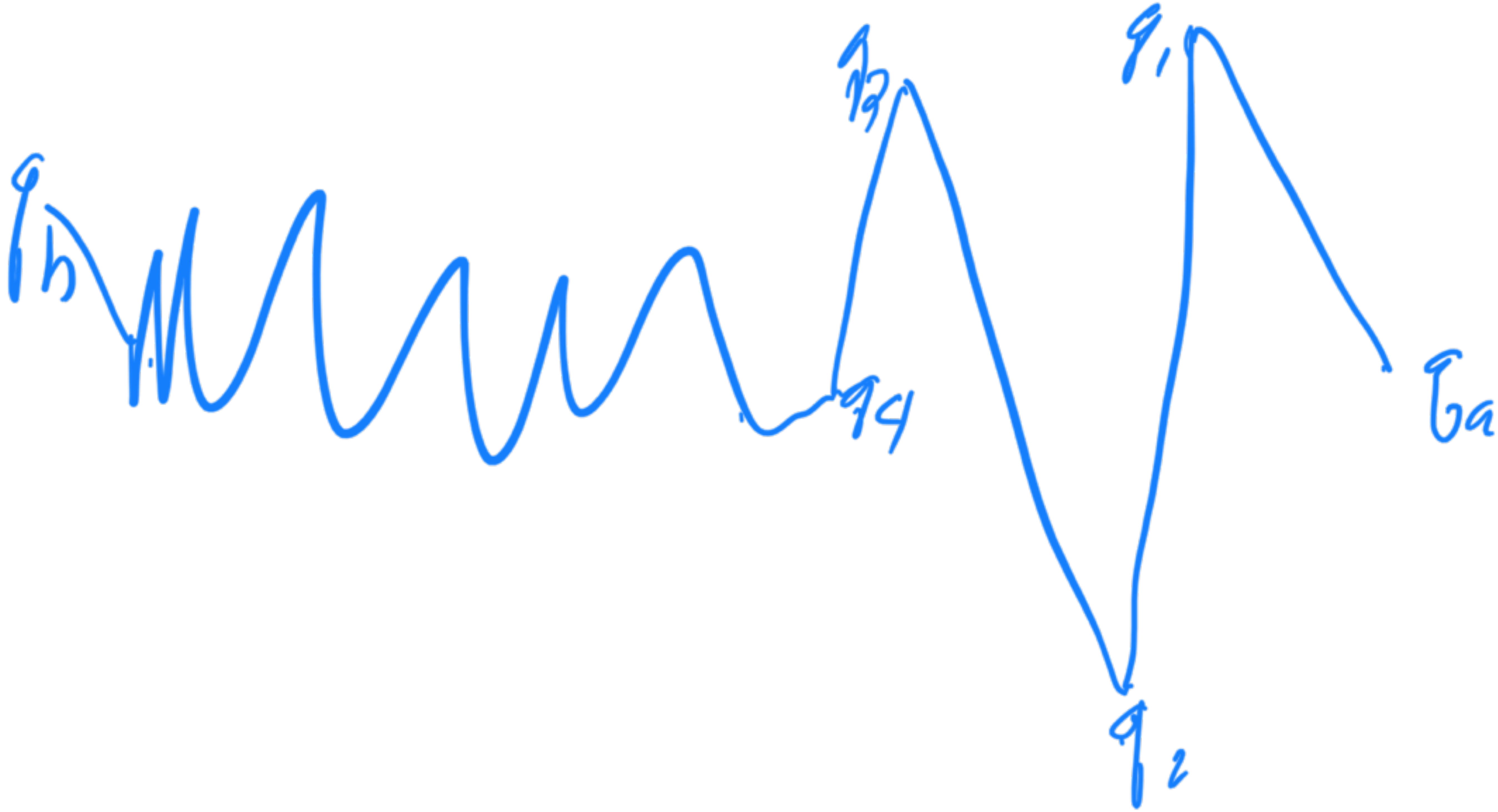
$$\dot{q}_a = n(q_1 - q_a) \quad \text{huge}$$

$$\langle q_b | e^{-iHt/\hbar} | q_a \rangle = (\ ) \int \exp\left\{ i \frac{1}{n\hbar} \sum_j \left[\frac{m \dot{q}_j^2}{2} - V(q_j) \right] \right\}$$

$$= (\ ) \int e^{\frac{i}{\hbar} \int L\,dt} \mathcal{D}q = (\ ) \int e^{iS/\hbar} \mathcal{D}q$$

$$\mathcal{D}q = \left(\frac{mn}{2\pi i \hbar (t_b - t_a)} \right)^{\frac{n}{2}} dq_1 dq_2 \cdots dq_{n-1}$$

$q_n \; q_{n-1} \; q_{n-2} \; \ldots \; q_2 \; q_1 \; q_a$



q

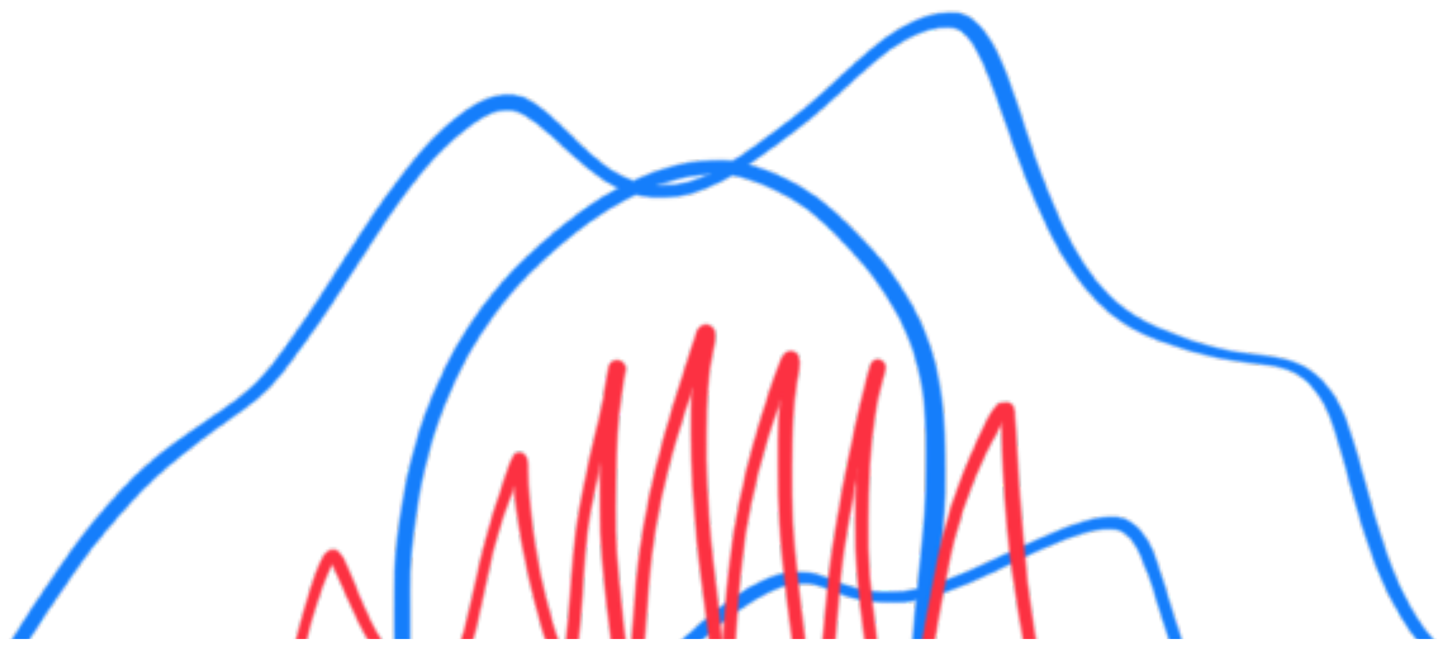
q_b

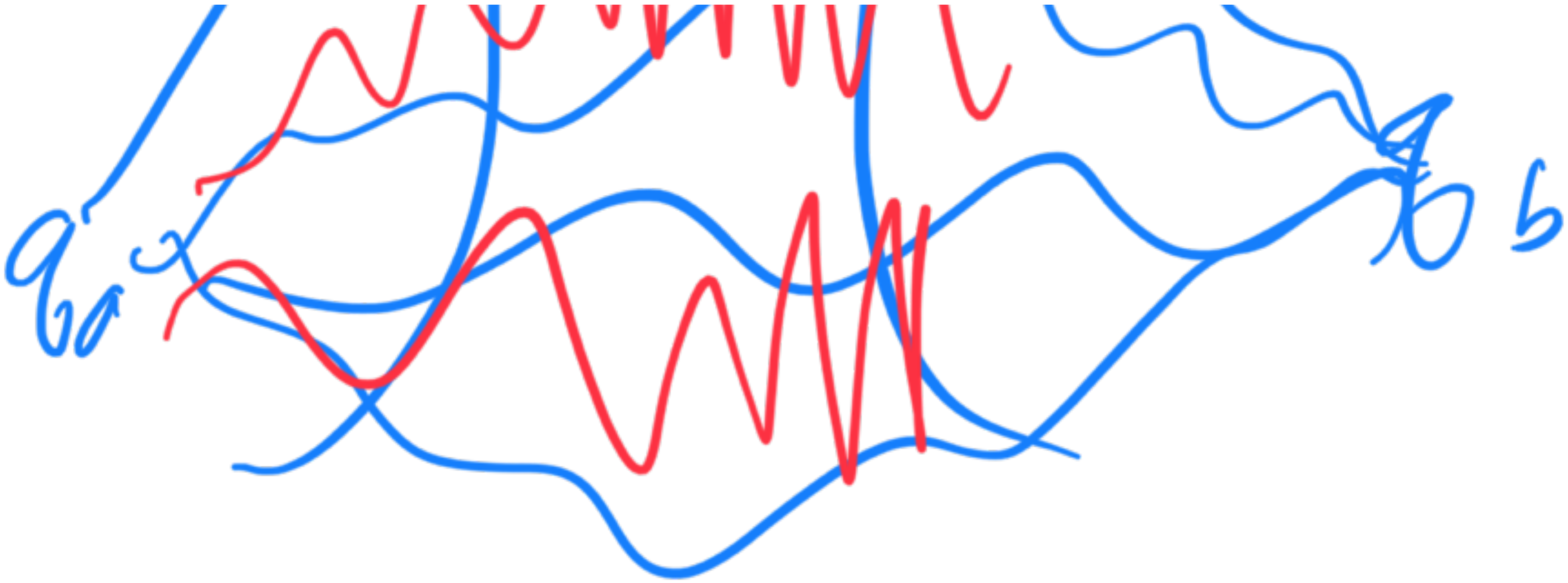
q_a

t

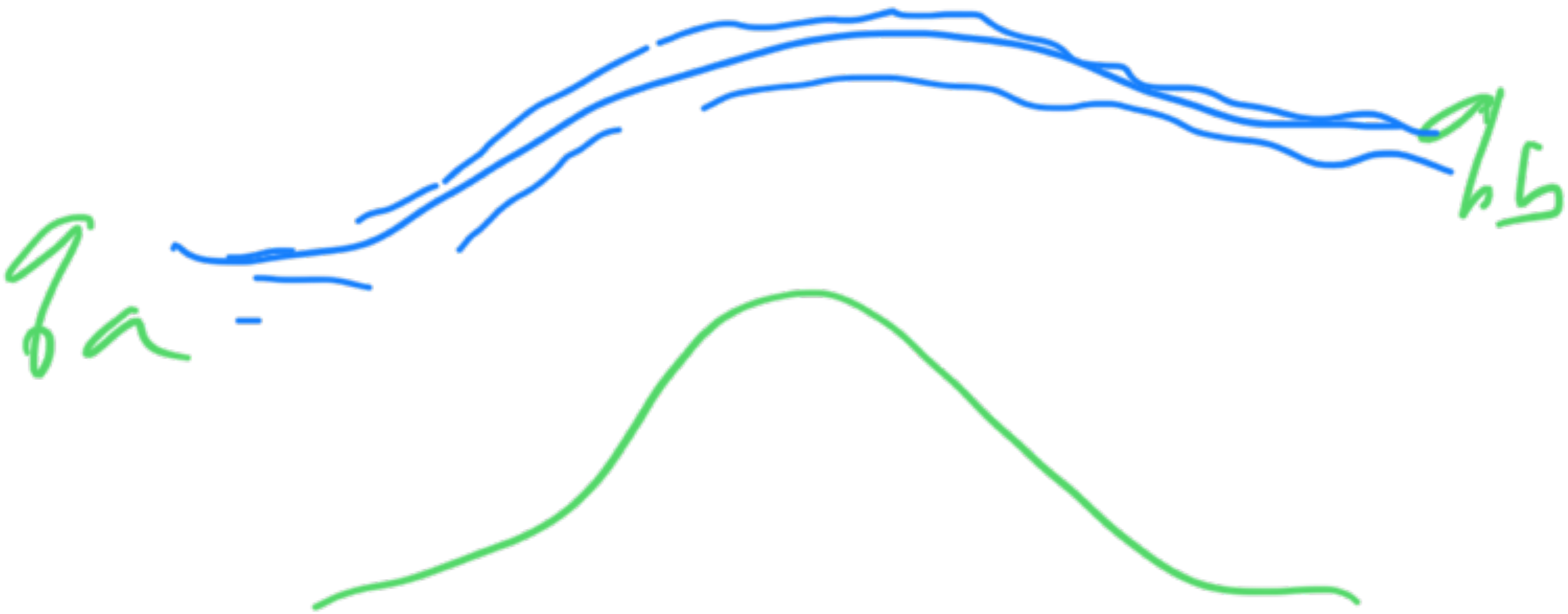
$$q(t') = \frac{t'}{t}(q_b - q_a) + q_a$$

$V(q)$

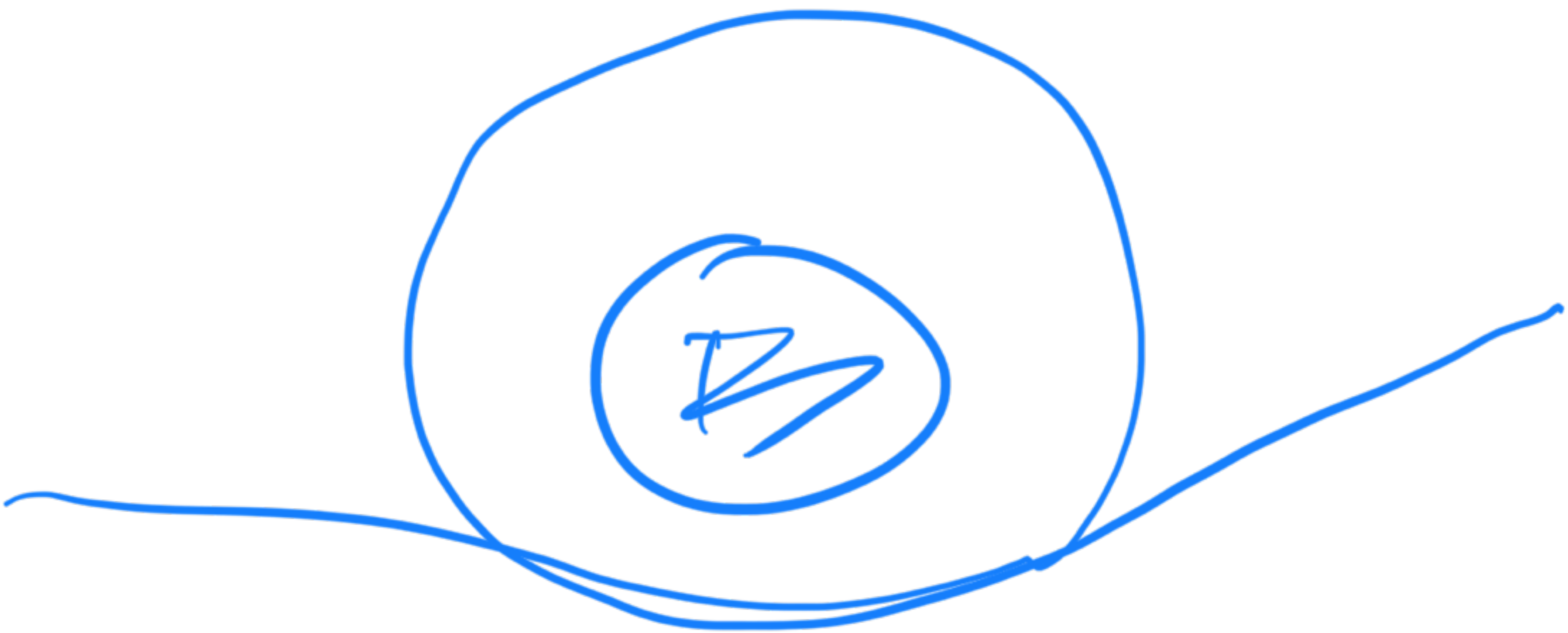


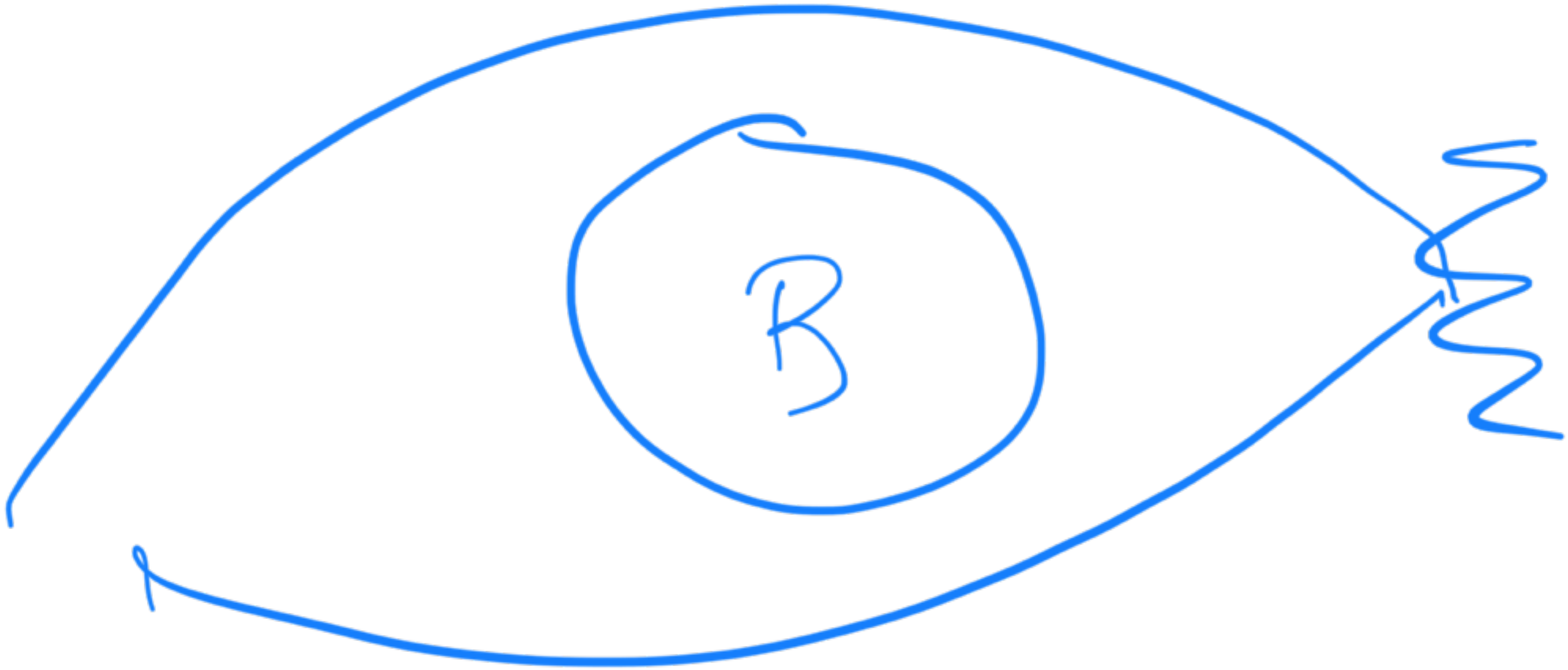


$$\langle q_b | q_a \rangle = \int e^{\frac{i}{\hbar} S} \, \mathcal{D}q$$



$$\int \frac{e}{c} \vec{A}\cdot\vec{v}\, dt$$





$$\rho^{\dagger} = \rho$$

$$\operatorname{tr} \rho = 1$$

$$\langle \psi | \rho | \psi \rangle \geq 0$$

$$Z(\beta) = \mathrm{tr}\, e^{-\beta H} = \mathrm{tr}\, e^{-\beta H}$$

$$\rho = \frac{e^{-\beta H}}{\mathrm{tr}\, e^{-\beta H}}$$

$$\mathrm{tr}\, e^{-\beta H} = \int dq_a \langle q_a | e^{-\beta H} | q_a \rangle$$

$$e^{-\beta H}$$

$$= \mathrm{Tr}\, e^{\ldots}$$

$$e^{-\beta H} \to |E_0\rangle\langle E_0|$$

$$\beta \to \infty$$

$$e^{-\beta H} = e^{-\beta H}\, I$$

$$= e^{-\beta H} \sum |E_n\rangle\langle E_n|$$

$$= \sum_n |E_n\rangle e^{-\beta E_n} \langle E_n|$$

$\beta \to \infty$ or $T \to 0$

$$e^{-\beta H} \longrightarrow |E_0\rangle e^{-\beta E_0} \langle E_0|$$


q