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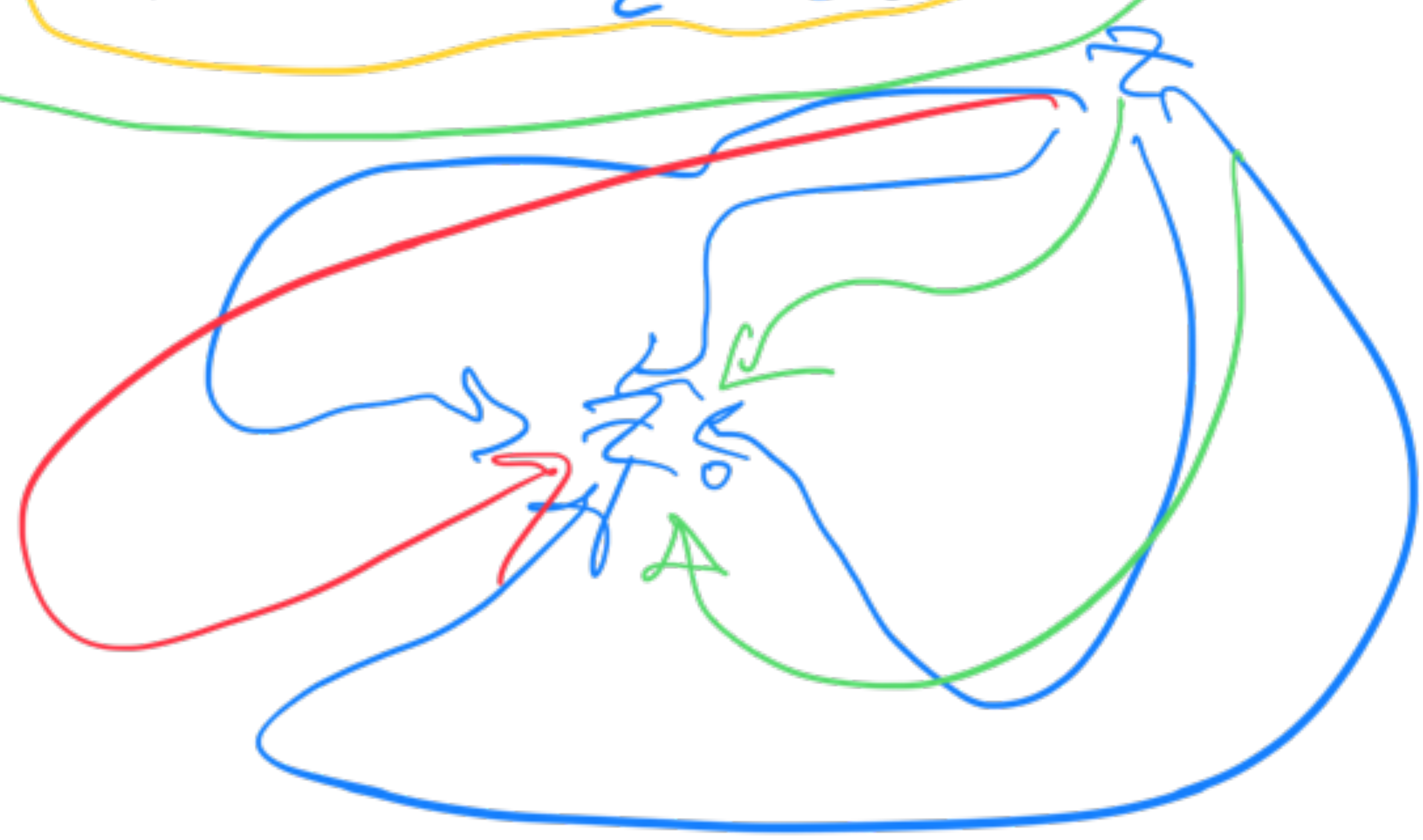
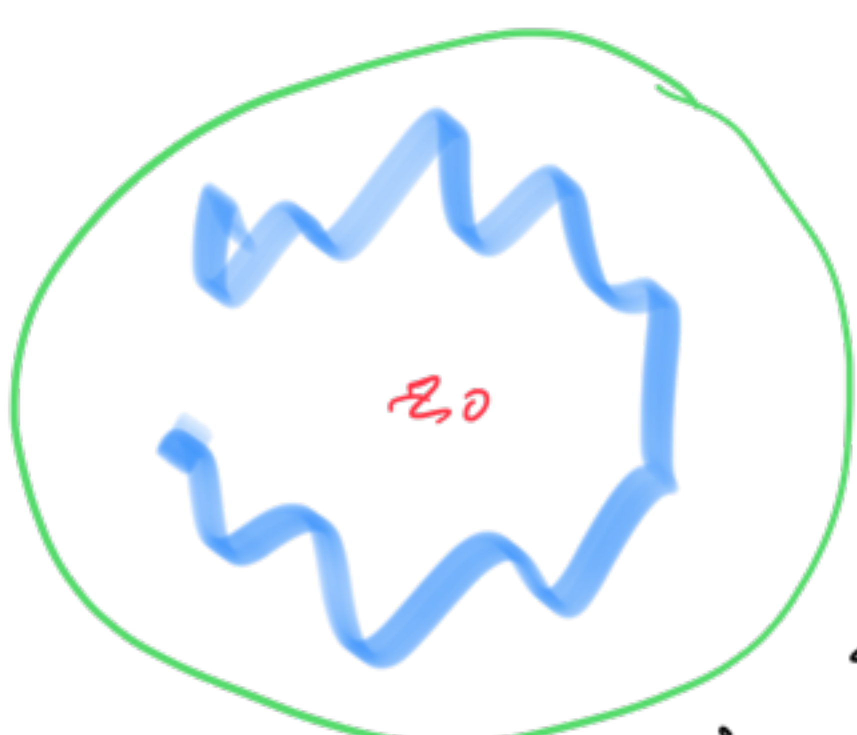
$z = x + iy$

$f(z) = u(x,y) + i\,v(x,y)$

$= u(z) + i\,v(z)$

$$f'(z_0) = \frac{f(z) - f(z_0)}{z - z_0}$$





analytic at z_0 and inside disk

$f = u + iv$

$\delta f(x,y) = \delta u(x,y) + i\,\delta v(x,y)$

$= u_x dx + u_y dy + i v_x dx + i v_y dy$

$= f'(z)\,dz = f'(z)(dx + i\,dy)$

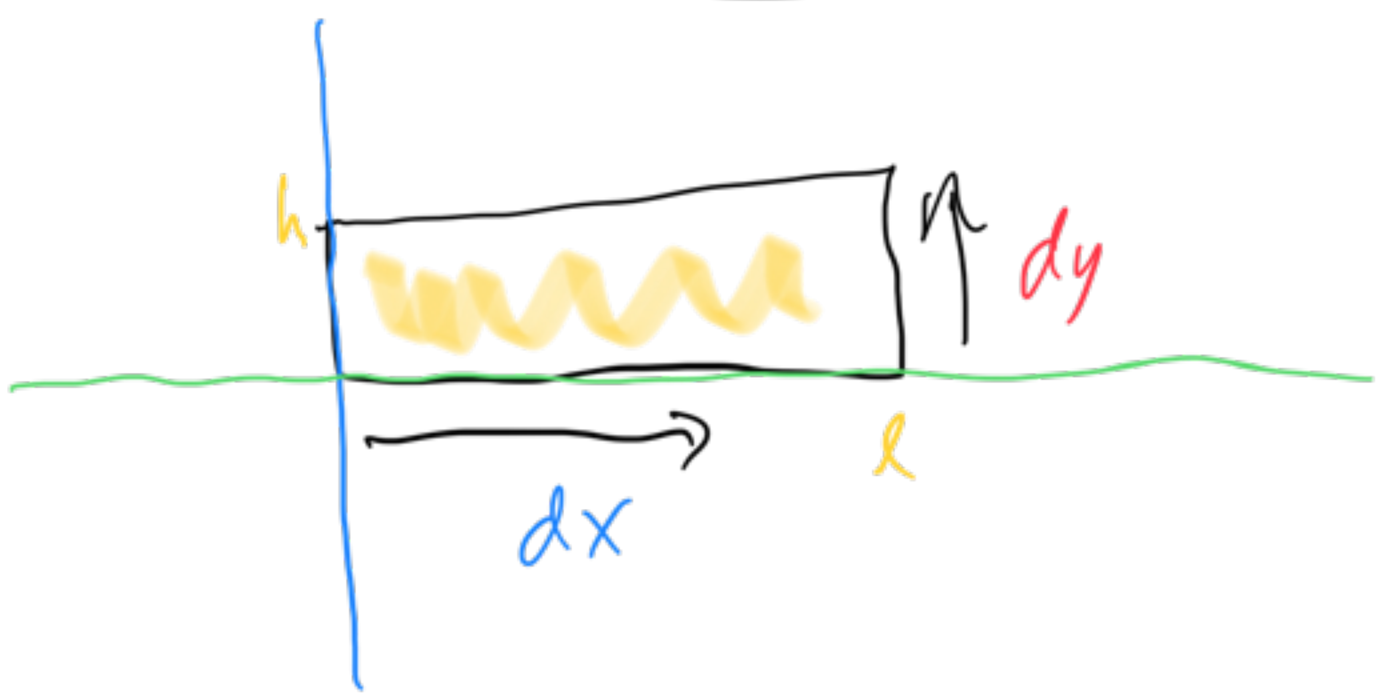
$$(u_x + i v_x) = f'$$

$$(u_y + i v_y) = i f'$$

$$i(u_x + i v_x) = u_y + i v_y$$

$$-v_x = u_y \qquad u_x = v_y$$

C-R conditions



$$\int_0^0 f(x,y)(dx + i\,dy)$$

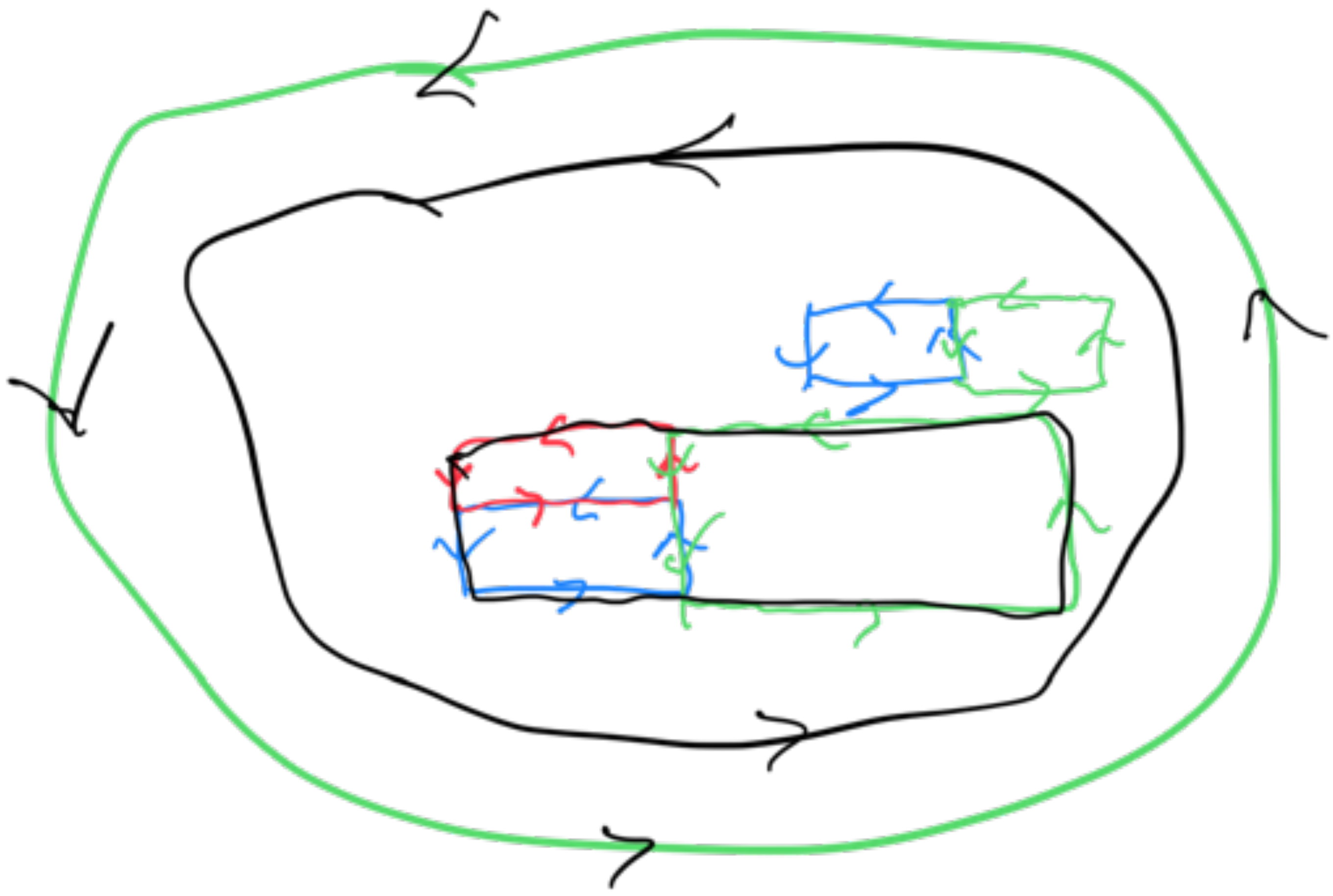
$$\mathrm{Re} \oint_R f(z)\,dz = -\int_0^l \int_0^h (u_y + v_x)\,dy\,dx = 0$$

$$\mathrm{Im} \oint_R f(z)\,dz = \int_0^l \int_0^h (-v_y + u_x)\,dy\,dx = 0$$

$$\oint_R f(z)\,dz = 0$$

if f analytic inside & on

rectangle R.



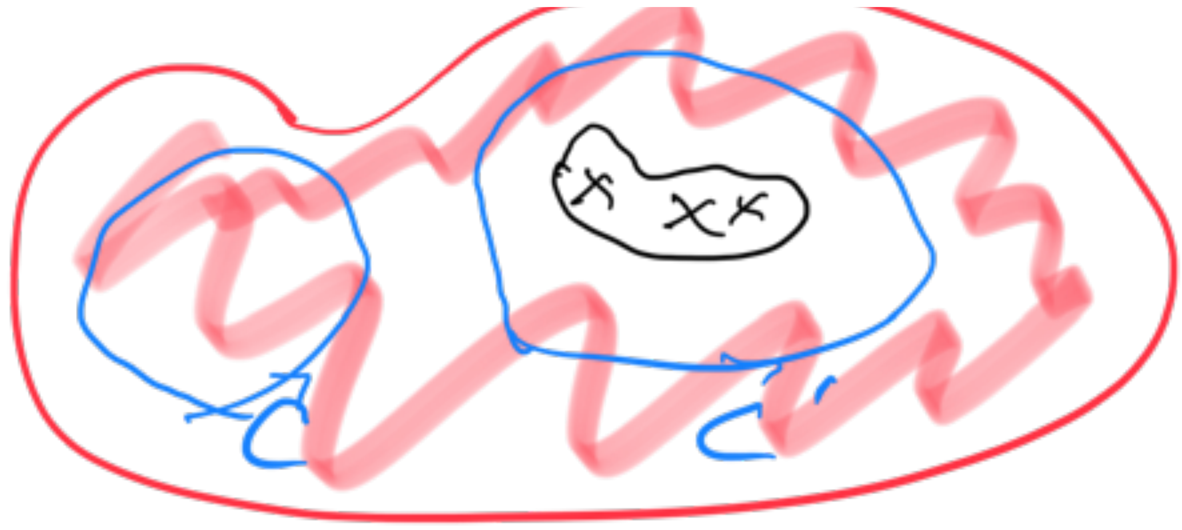
$$\oint_C f(z)\,dz = 0$$

if $C \subset R$

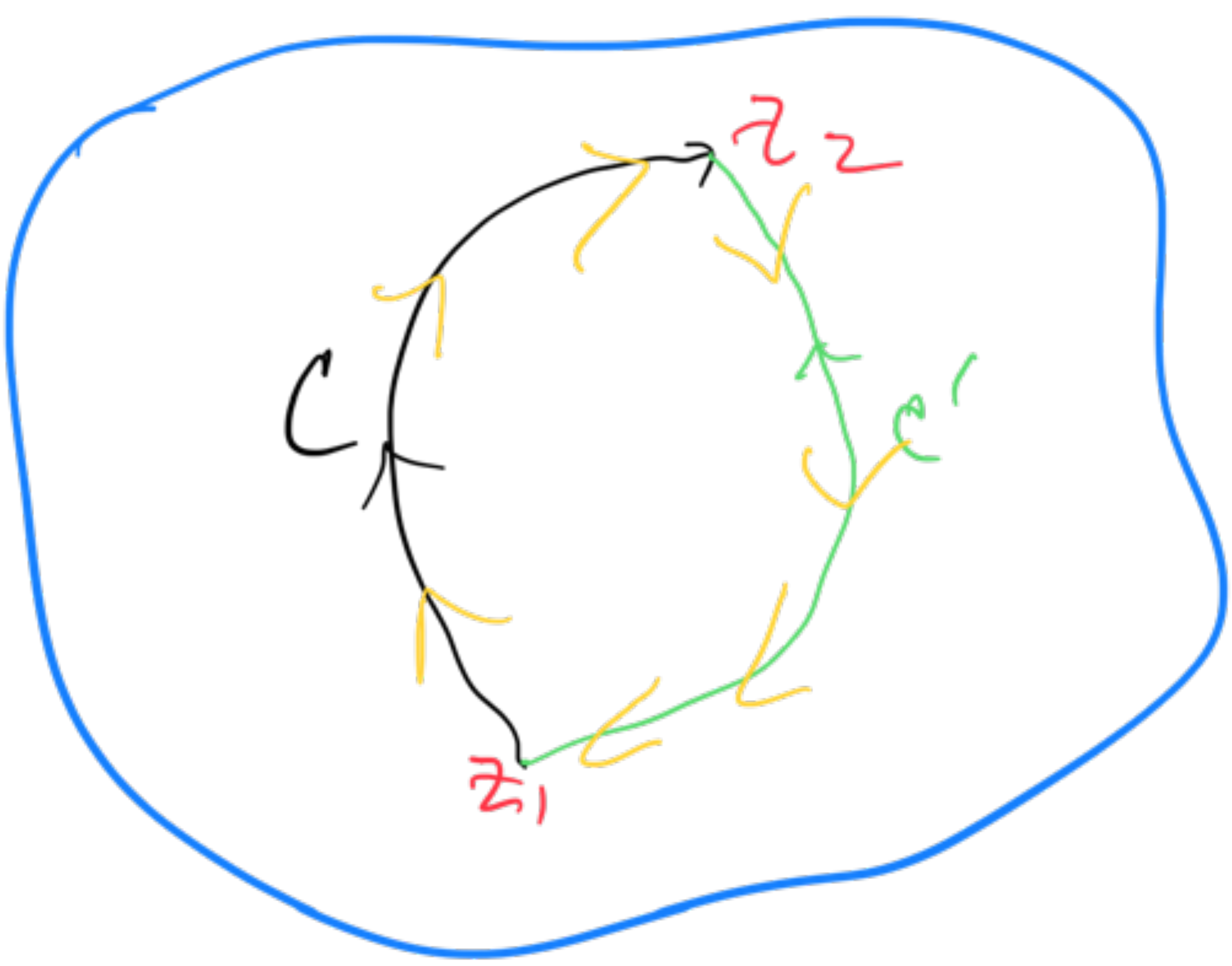
R has no holes

$\oint_C f(z)\,dz = 0$

closed contour = loop



$$\oint_C f\,dz = 0 \qquad \oint_{C'} f\,dz \neq 0$$

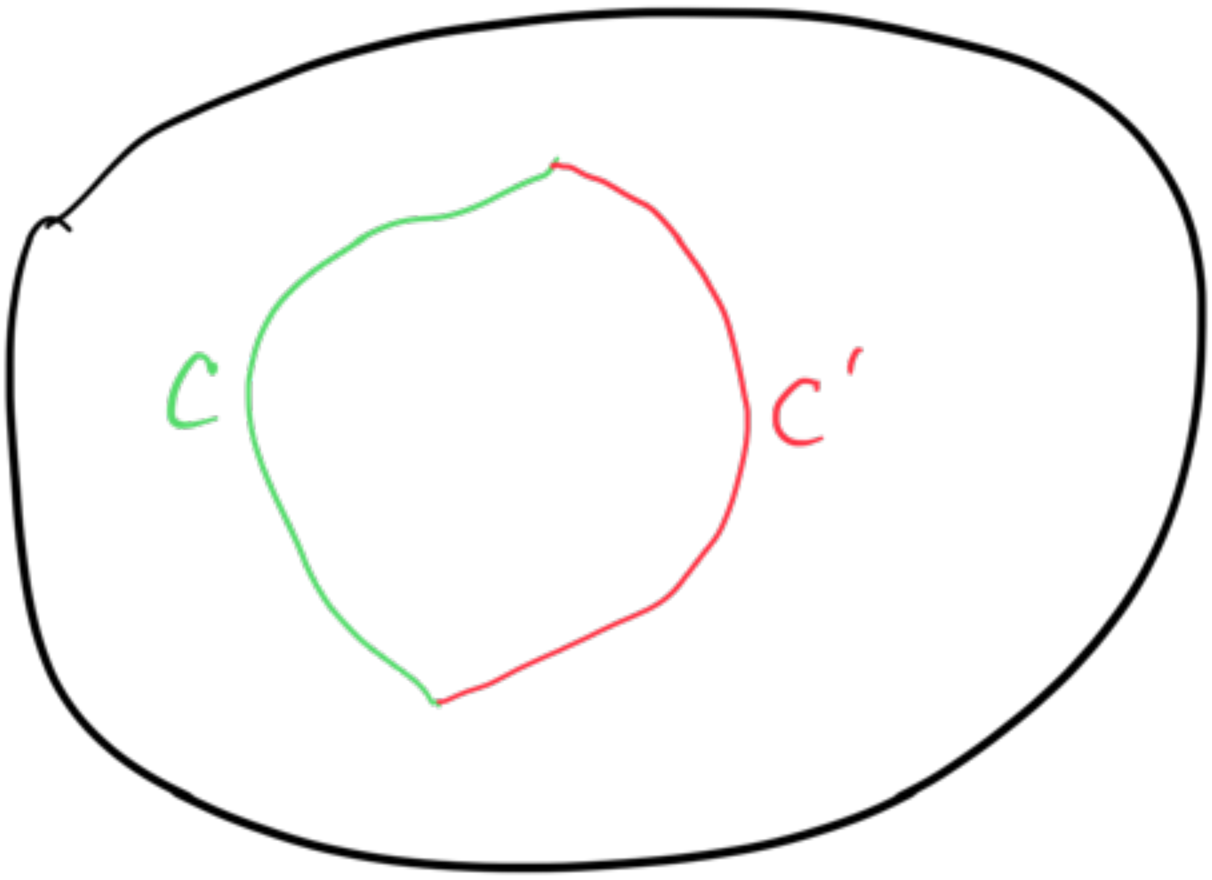


$$\int_C f(z)\,dz = \int_{C'} f(z)\,dz$$

$$\int_{C-C'} f(z)\,dz) = \oint f(z)\,dz = 0$$

$$= \int_C f(z)\,dz - \int_{C'} f(z)\,dz = 0$$

$$\int_C f(z)\,dz = \int_{C'} f(z)\,dz$$



$$\oint_\epsilon \frac{f(z)}{z-z_0}\,dz = \oint \frac{f(z_0) + i f'(z_0)(z-z_0)}{z-z_0}\,dz$$

$$z - z_0 = \epsilon e^{i\theta} \qquad dz = i\epsilon e^{i\theta}\,d\theta$$

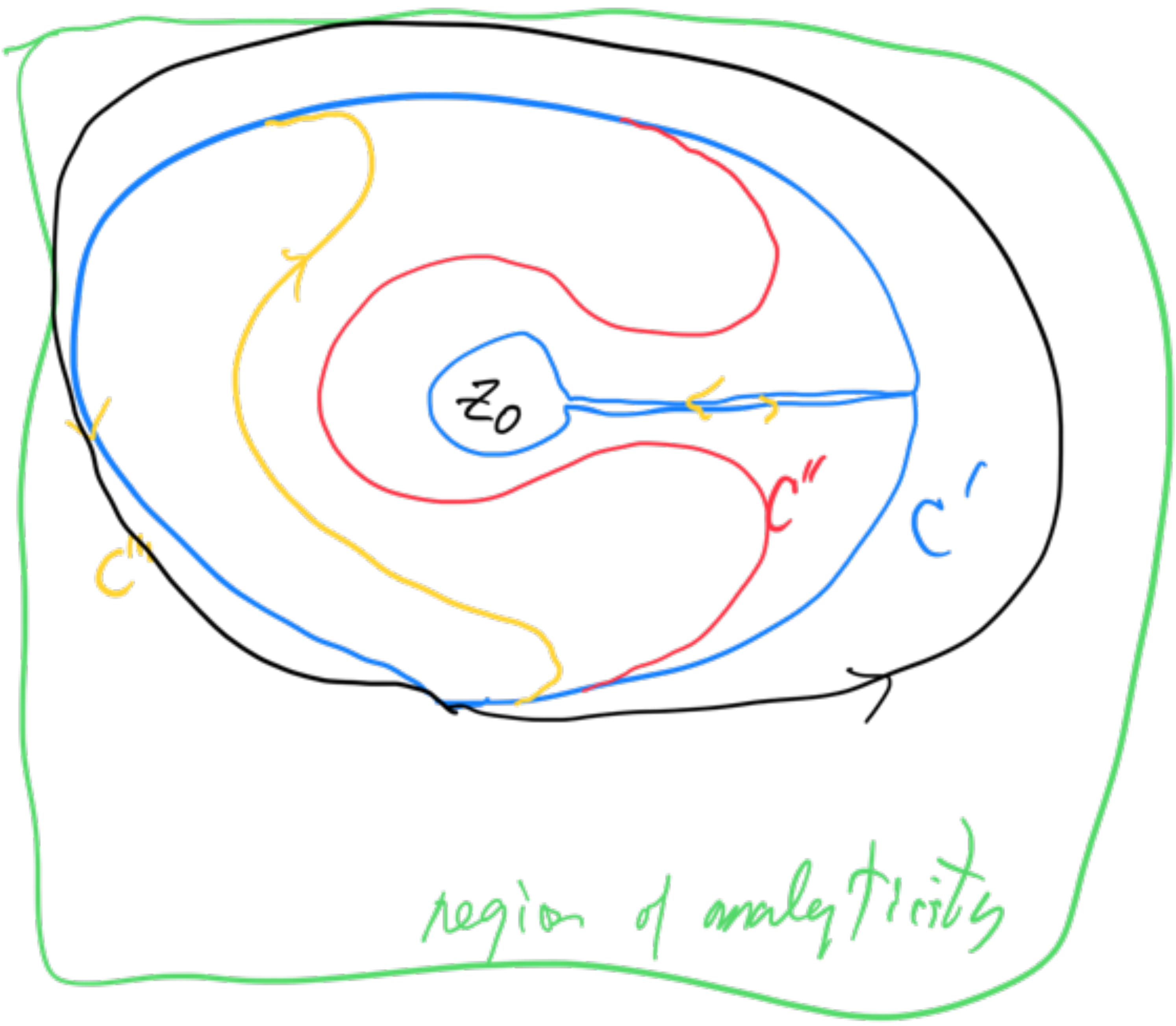
$$\int_0^{2\pi} \frac{\{f(z_0) + i f(z_0)\epsilon e^{i\theta}\}\, i \cancel{\epsilon} \cancel{e^{i\theta}}\,d\theta}{\cancel{\epsilon} \cancel{e^{i\theta}}}$$

$$= 2\pi i f(z_0)$$

$$\int_0^{2\pi} e^{i\theta}\,d\theta = 0$$

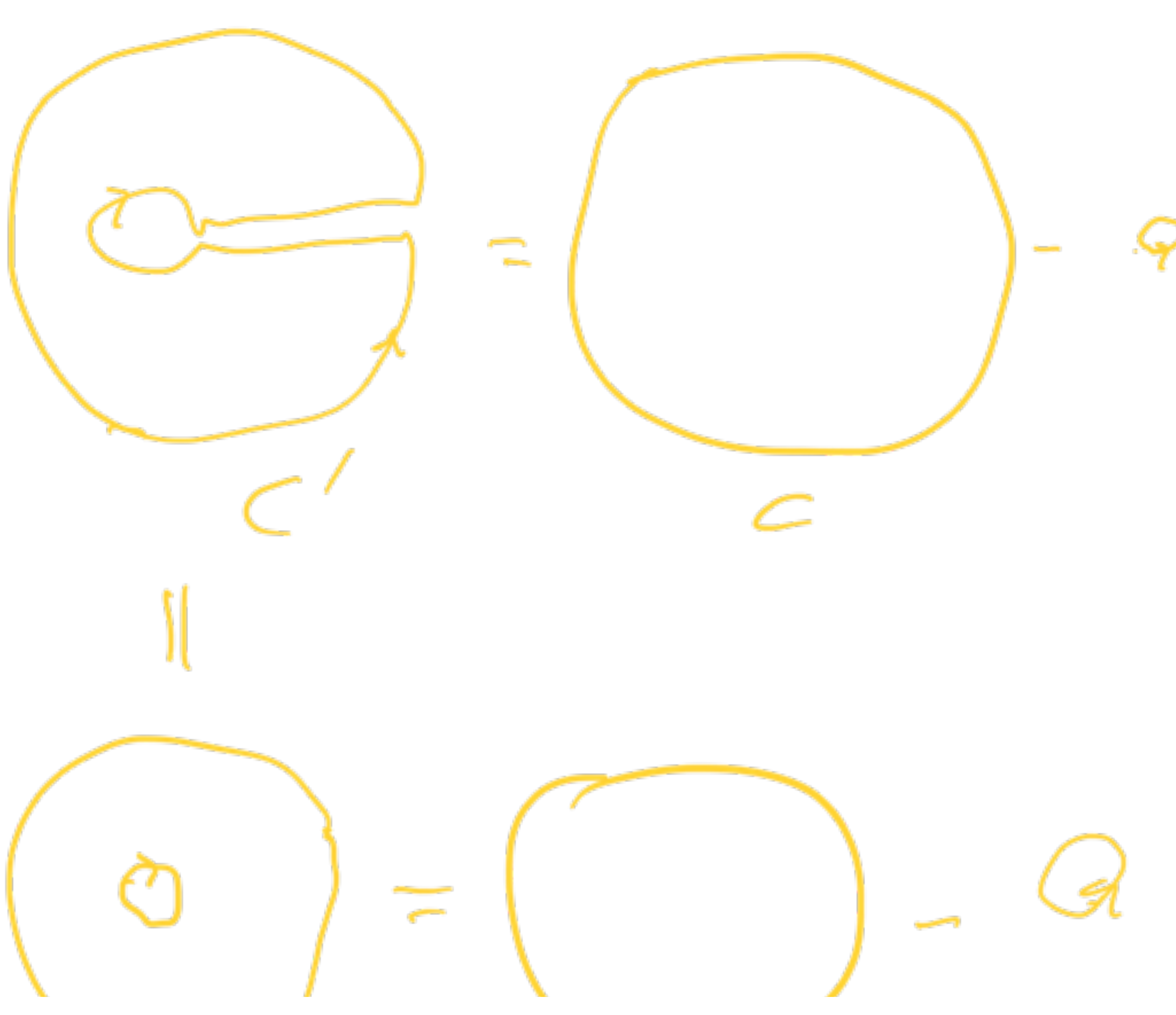
$$\frac{1}{\quad} \oint \frac{f(z)}{z-z_0}\,dz$$

$f(z_0) \quad 2\pi i \int_{\epsilon} \quad z - z_0$



$$0 = \oint_{C'} \frac{f(z)\,dz}{z - z_0} = \oint_{C''} \frac{f(z)}{z - z_0}\,dz = \oint_{C'''} \frac{f(z)}{z - z_0}\,dz$$

$$= \int_C \frac{f(z)}{z - z_0}\,dz - \int_{\epsilon} \frac{f(z)\,dz}{z - z_0}$$



$$\oint_C \frac{f(z)}{z-z_0}\,dz = \oint_\epsilon \frac{f(z)}{z-z_0}\,dz$$

$$= 2\pi i\, f(z_0)$$

$$\oint \frac{f(z)}{z-z_0}\,dz = 2\pi i\, f(z_0)$$

$$\oint f(z)\,dz = 0$$

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$$f(x,y) = u(x,y) + i v(x,y)$$

C-R: f analytic iff

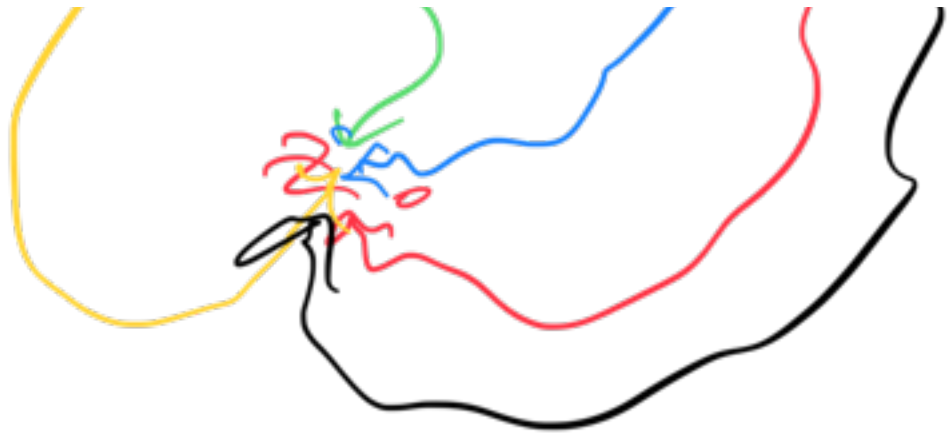
$$u_x = v_y \quad \& \quad \frac{\partial u}{\partial y} = u_y = -v_x = -\frac{\partial v}{\partial x}$$

C-R $\Rightarrow$ $u_{xx} + u_{yy} = 0$

$v_{xx} + v_{yy} = 0$

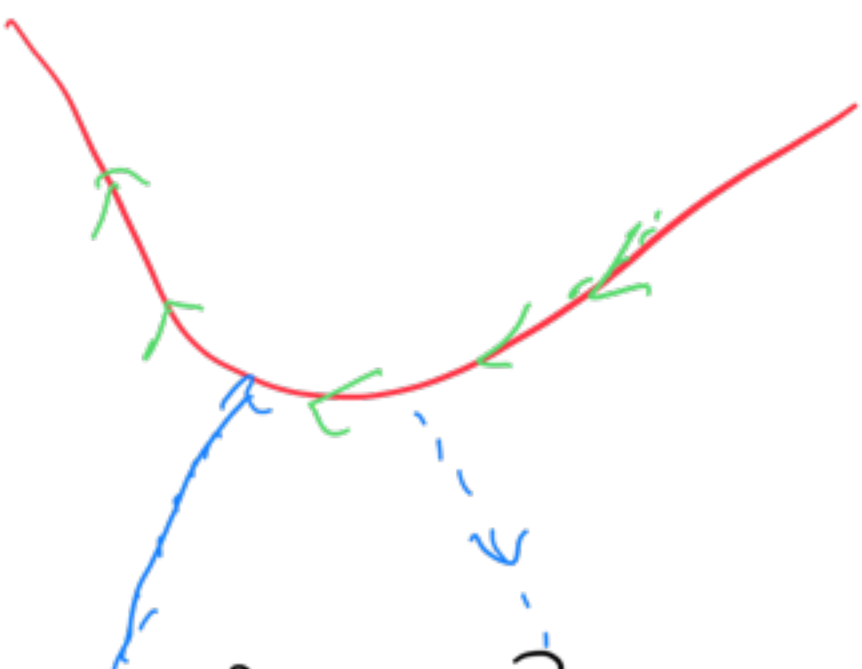
$\nabla \cdot E = -\nabla \cdot \nabla \phi = 0$ empty space

$$\frac{f(z)-f(z_0)}{z-z_0} \longrightarrow f'(z_0)$$



$$u_{xx} = v_{yx} = v_{xy} = -u_{yy}$$

$$\left.\begin{aligned} u_{xx} + u_{yy} &= 0 \\ v_{xx} + v_{yy} &= 0 \end{aligned}\right\} \text{harmonic}$$



$$\vec{E} = -\vec{\nabla}\phi$$

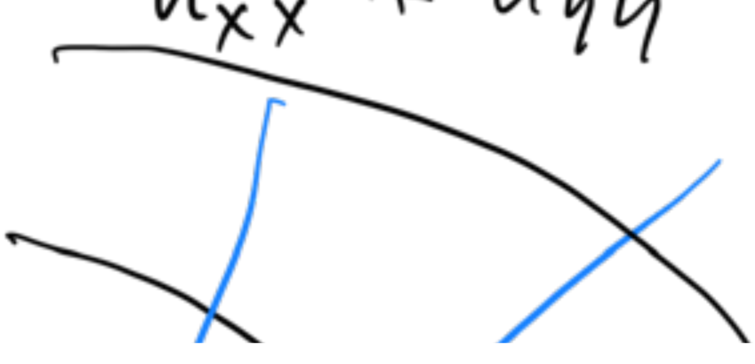
$$\nabla \cdot E = -\nabla \cdot \nabla \phi = \frac{\rho}{\epsilon_0}$$

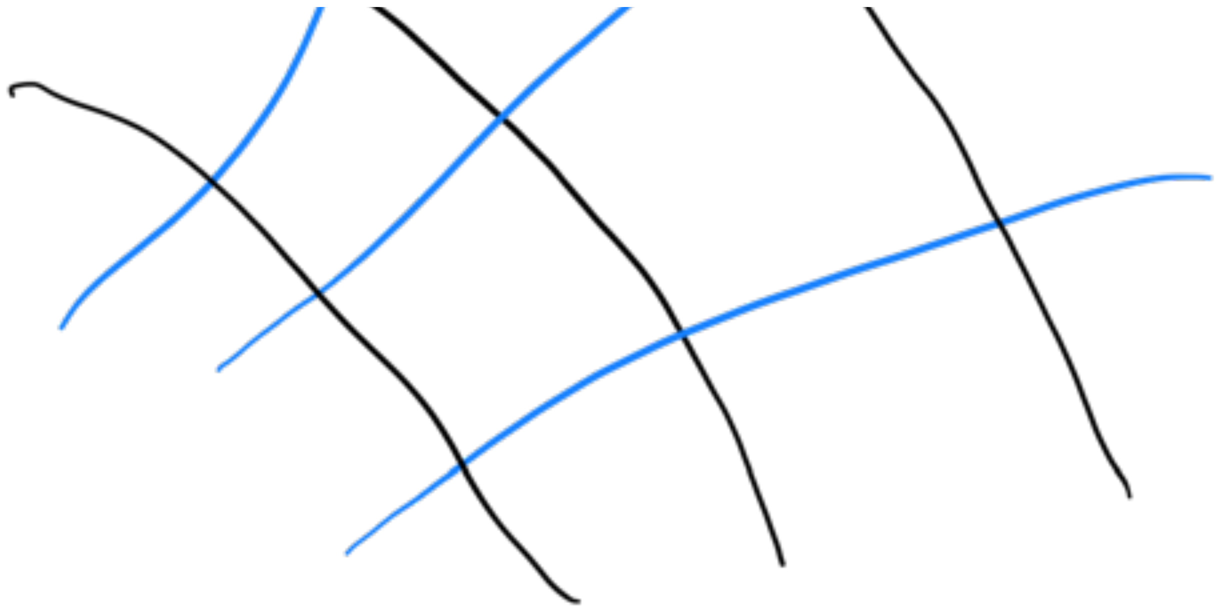
in empty space

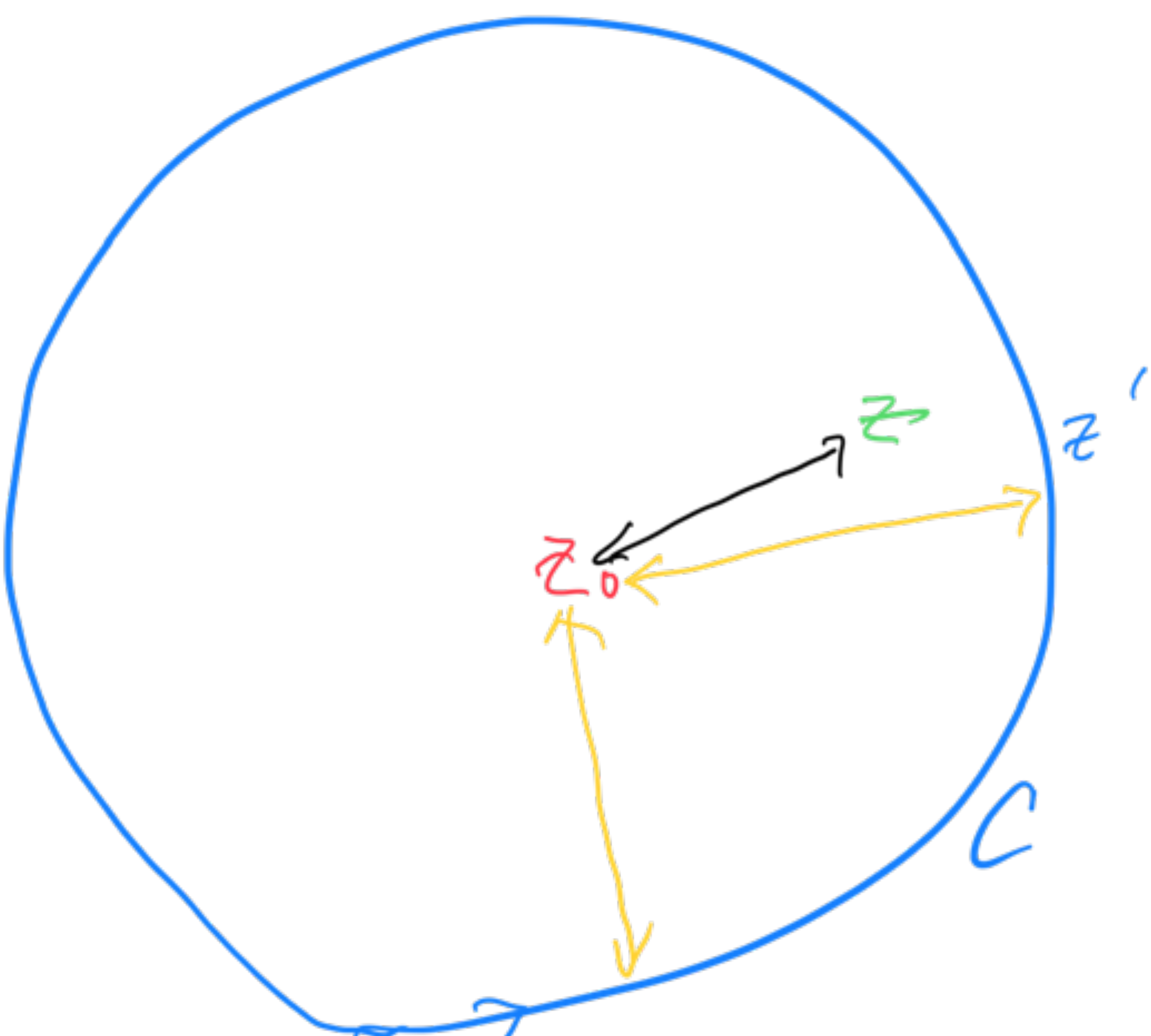
$$\nabla \cdot E = -\Delta \phi = 0$$

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0$$

$$u_{xx} + u_{yy} = 0$$







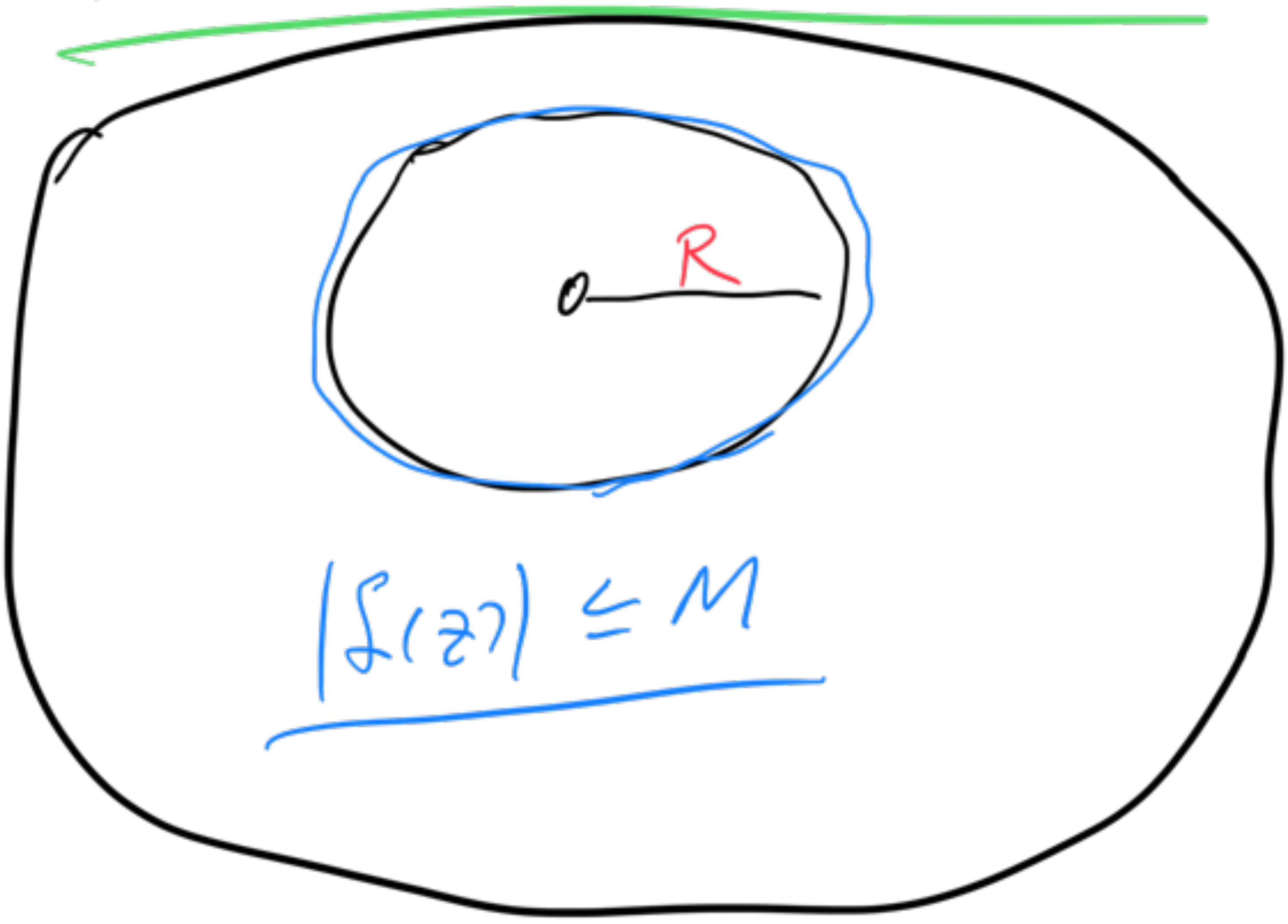
$$f(z) = \frac{1}{2\pi i} \oint_C \frac{f(z')\, dz'}{(z'-z_0)\left(1 - \frac{z-z_0}{z'-z_0}\right)}$$

$$\frac{1}{1-\epsilon} = \sum_{n=0}^{\infty} \epsilon^n$$

$$f^{(n)}(z_0) = \frac{n!}{2\pi i} \oint \frac{f(z')\, dz'}{(z'-z_0)^{n+1}}$$

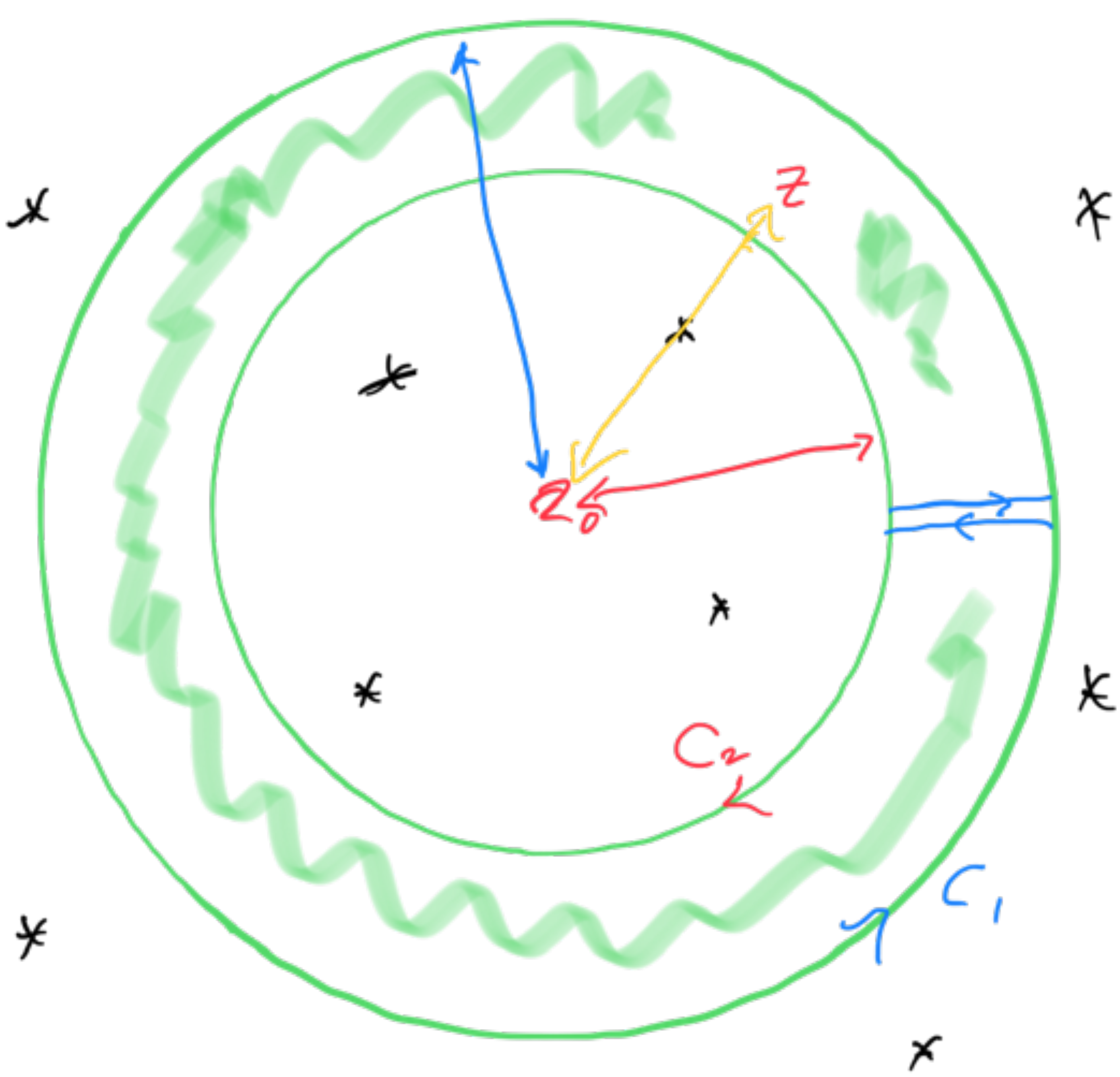
$$f(z) = \sum^{\infty} (z-z_0)^n \, f^{(n)}(z_0)$$

$n=0 \qquad\qquad n!$



$P_n(z) = z^n$

$z=0$ is root of order n



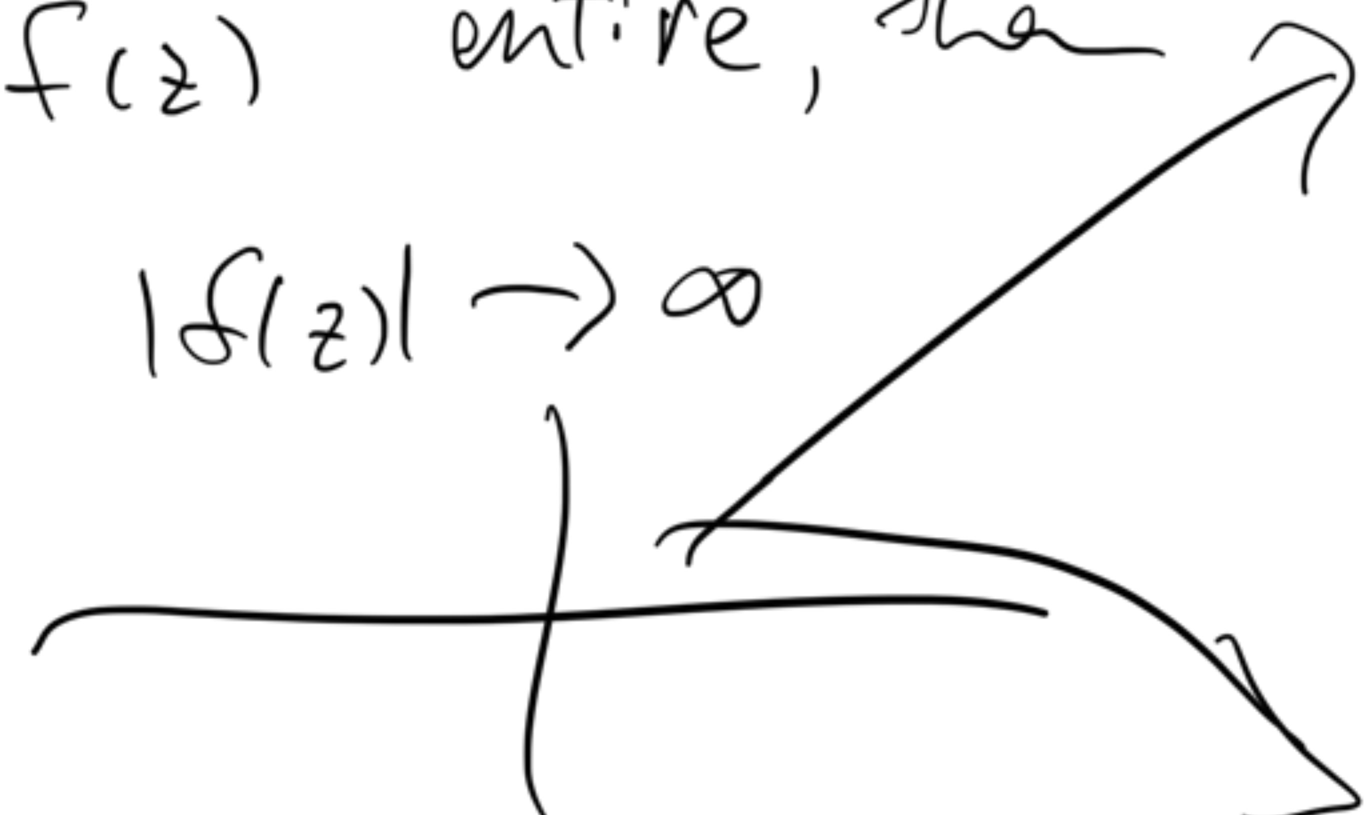
$$\left|\frac{z - z_0}{z - z_0}\right| < 1 \qquad \left|\frac{z - z_0}{z' - z_0}\right| < 1$$

$$\sum_{n=0}^{\infty} \epsilon^n = \frac{1}{1-\epsilon}$$

$n = -m - 1$ $\qquad$ $m+1 = -n$

If $f(z)$ entire, then

$$|f(z)| \to \infty$$

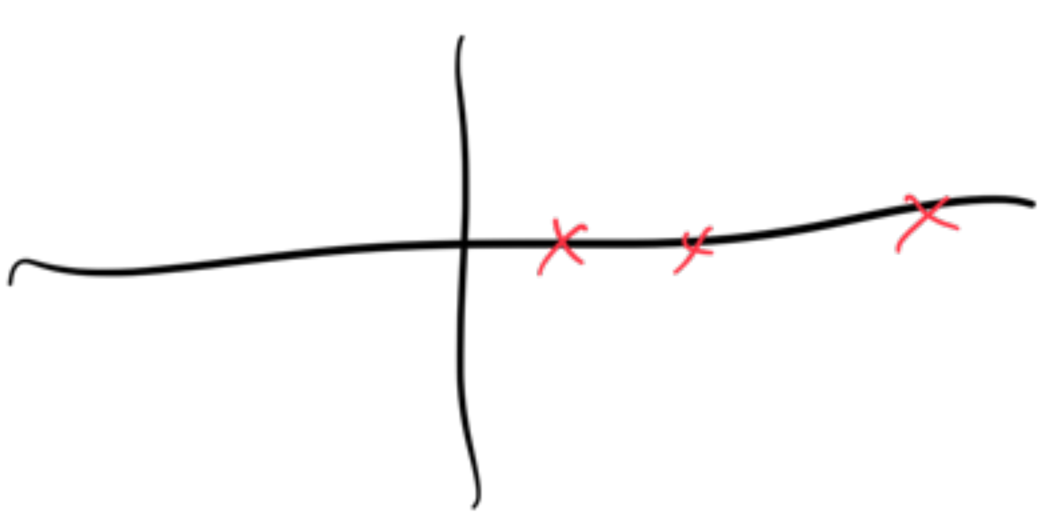


$|z| = \infty$ is "point at infinity"

$$f(z) = \frac{1}{z-1}$$

$$f(z) = \frac{c}{(z - z_0)^n}$$

$$f(z) = \frac{1}{(z-1)(z-2)(z-4)}$$



$$e^{-\frac{1}{z}}\ ,\ e^{\frac{3}{z}}$$

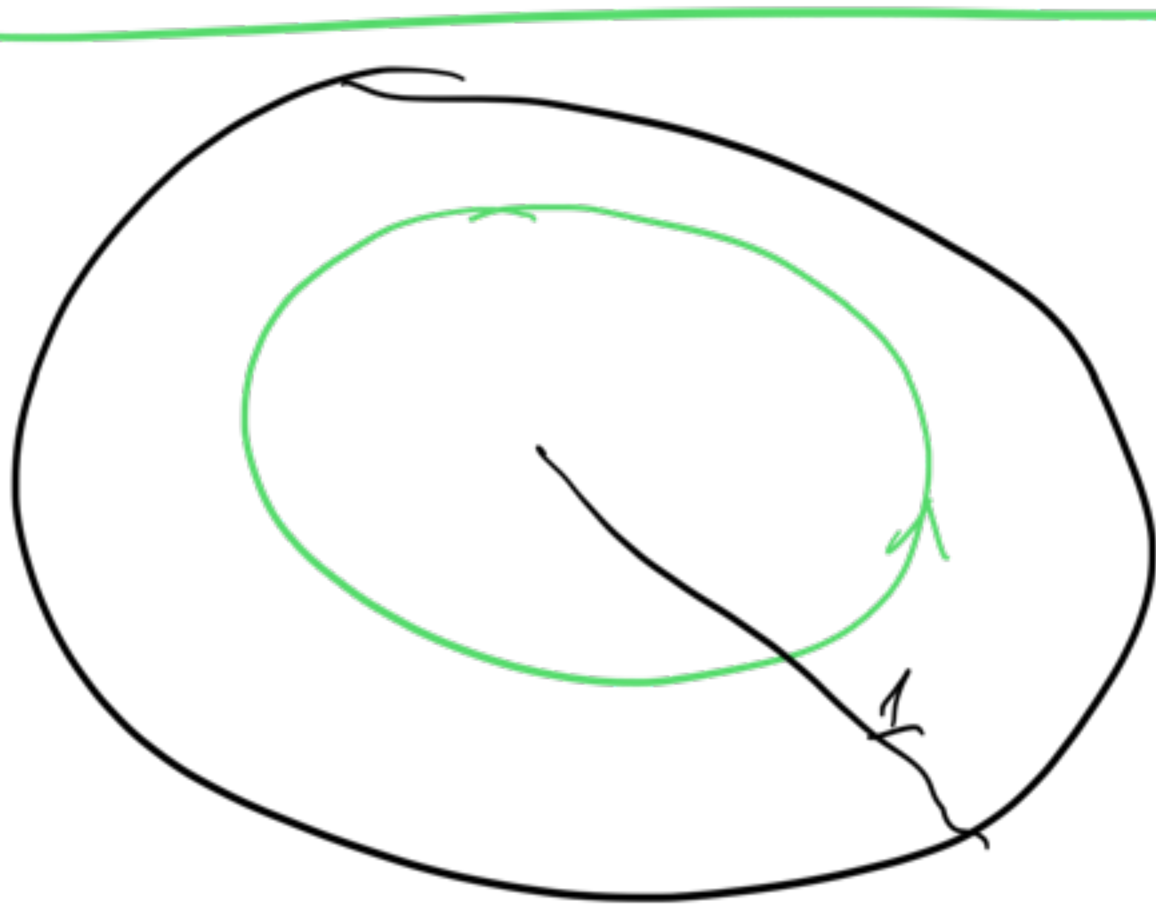
$$e^{\frac{2}{z^2}}$$

$$e^{\frac{1}{z}} = w$$

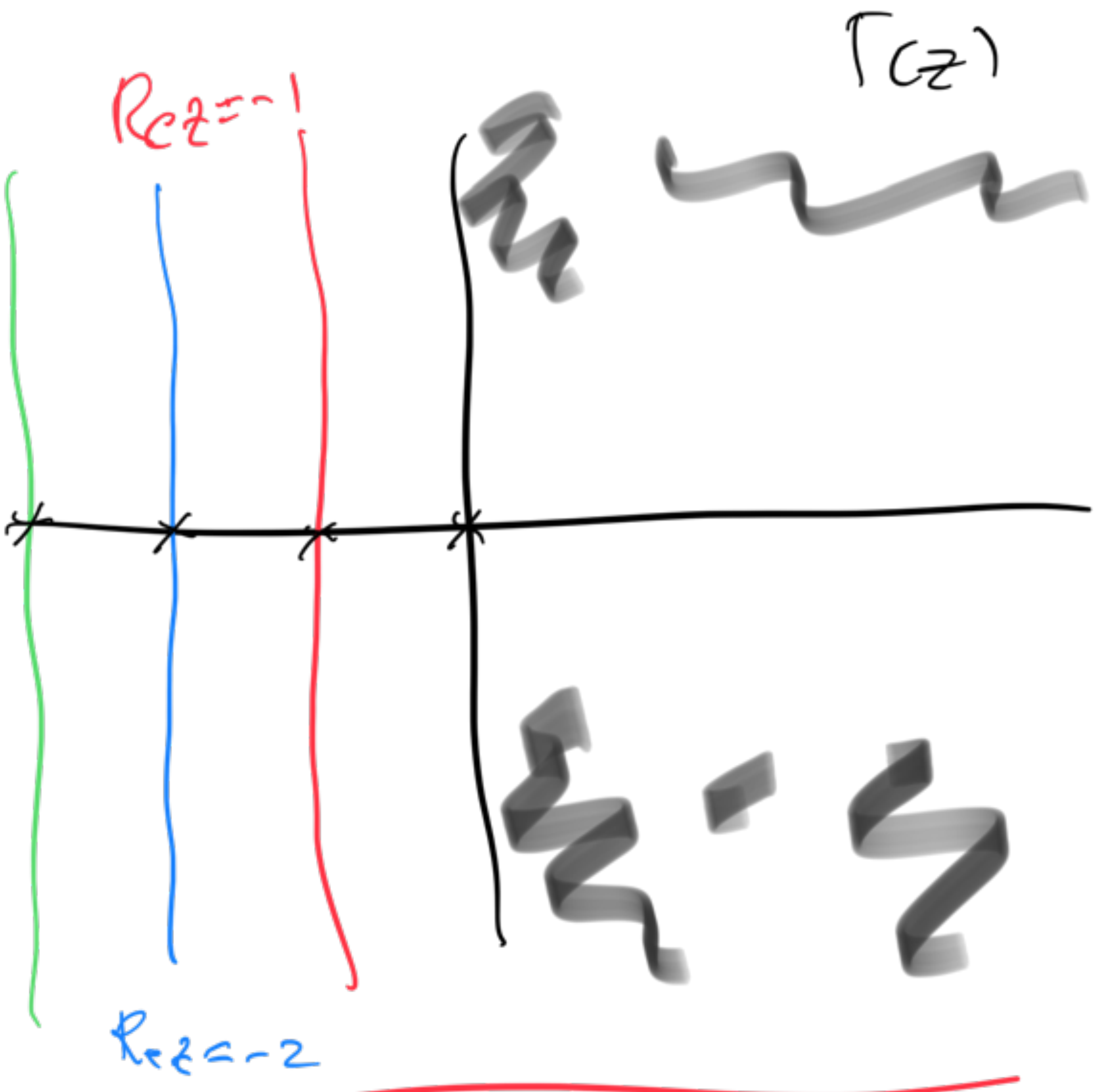
$$\log w = \frac{1}{z} + 2\pi n i$$

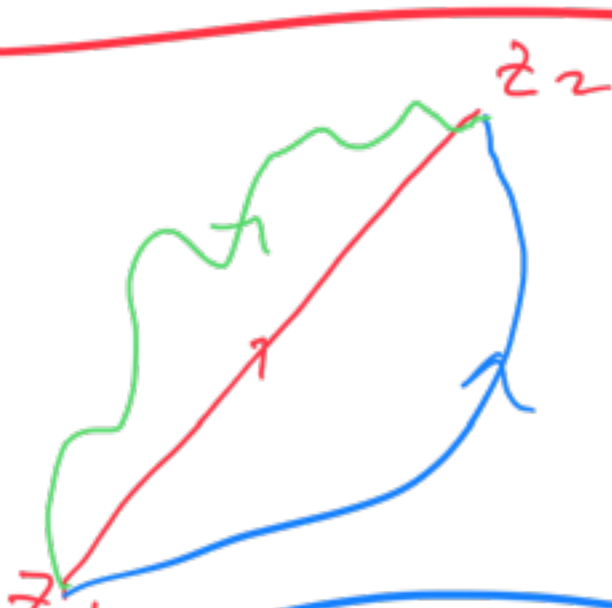
$$z = \frac{1}{\log w}$$

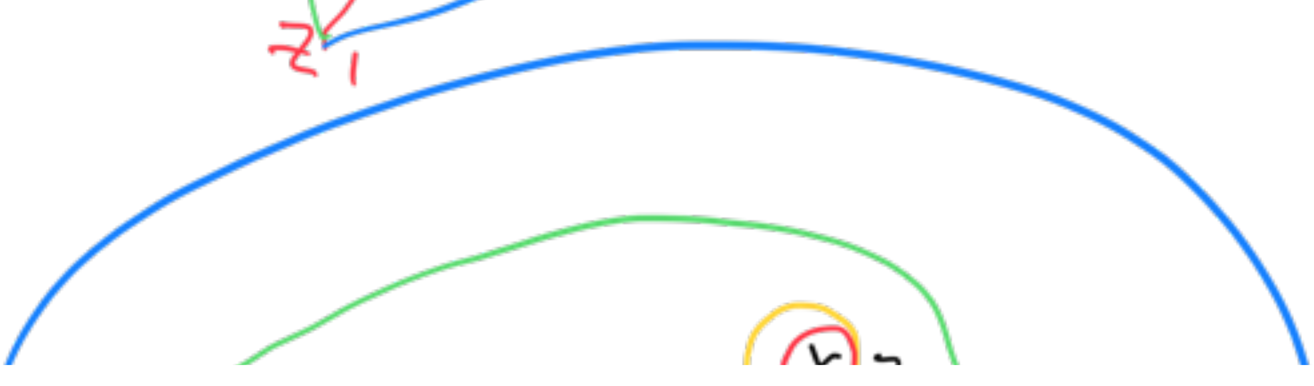
$$z_n = \frac{1}{\log w + 2\pi n i}$$

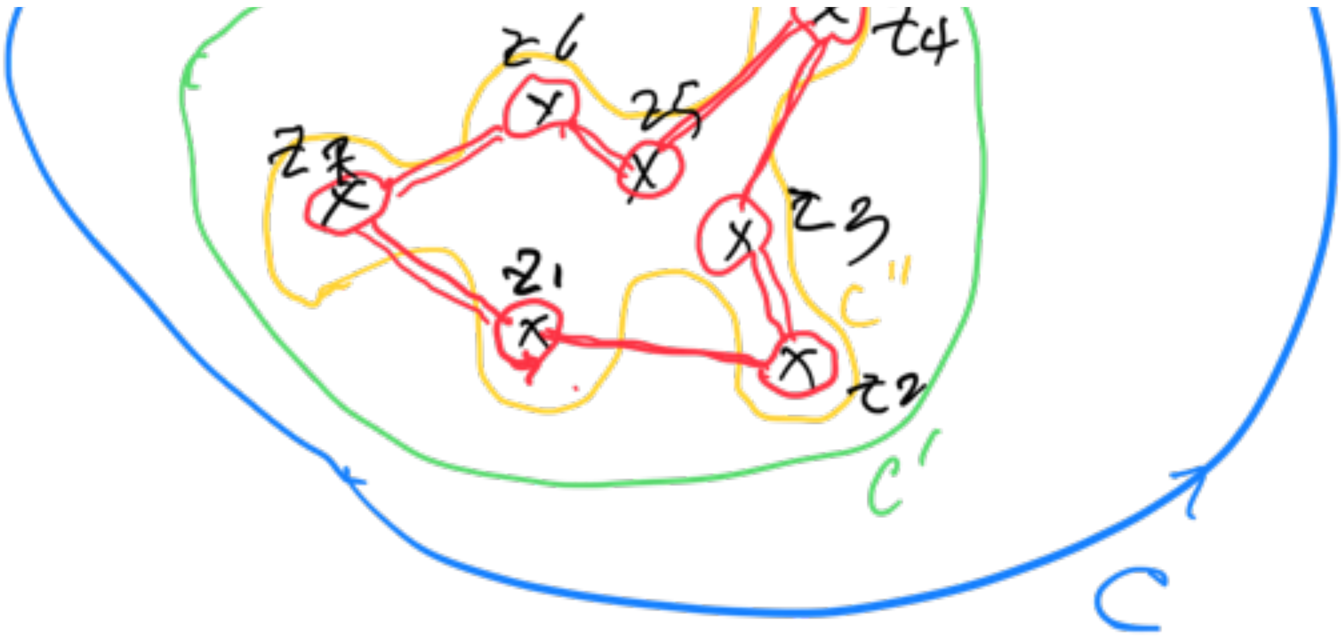


$$f(z) = \sum \frac{(z-z_0)}{m!} + \ldots$$







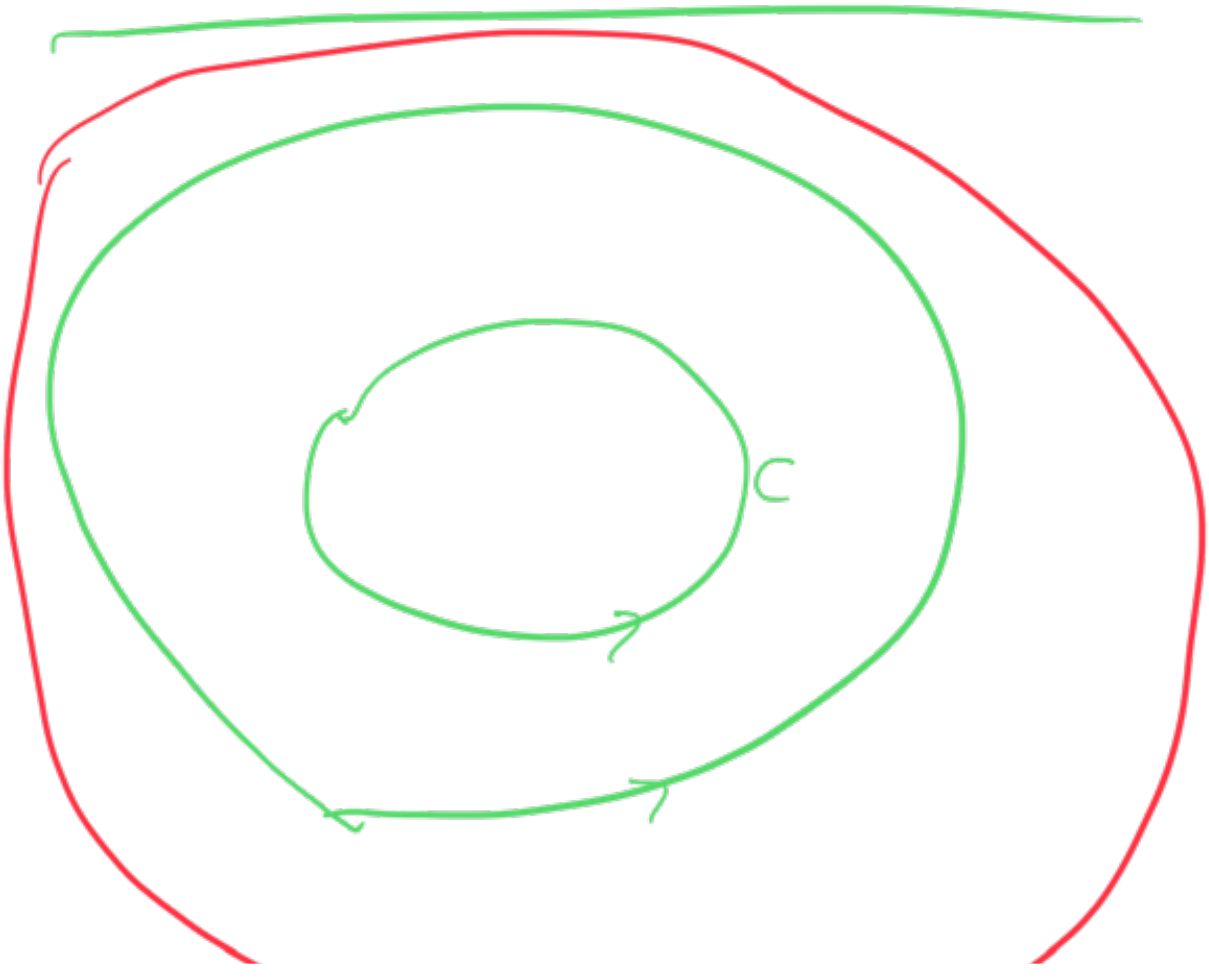


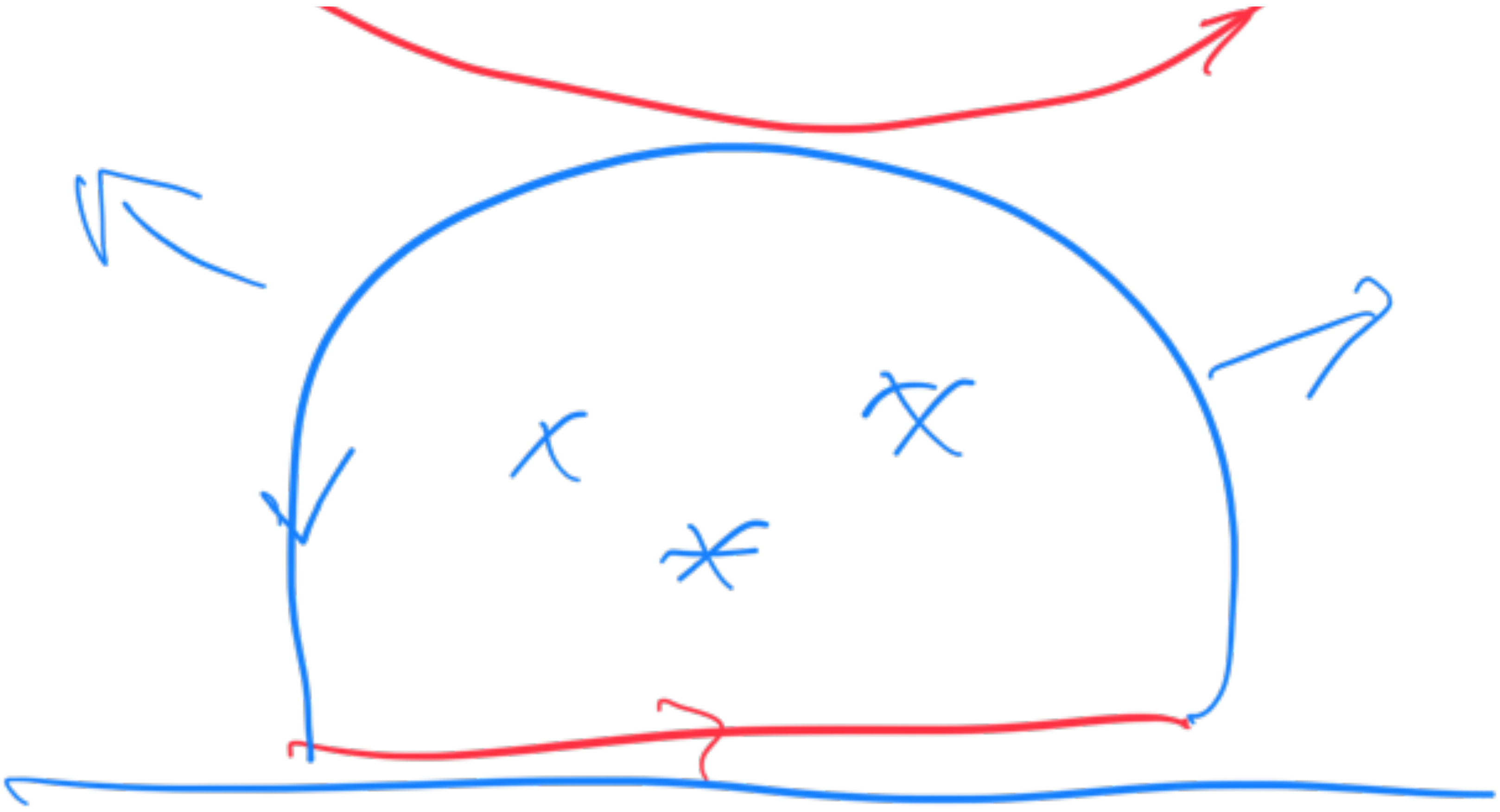
$$\oint_C f(z)\,dz = \oint_{C'} f(z)\,dz = \oint_{C''} f(z)\,dz$$

$$= \sum_{k=1}^{n} \oint_{C_k} f(z)\,dz$$

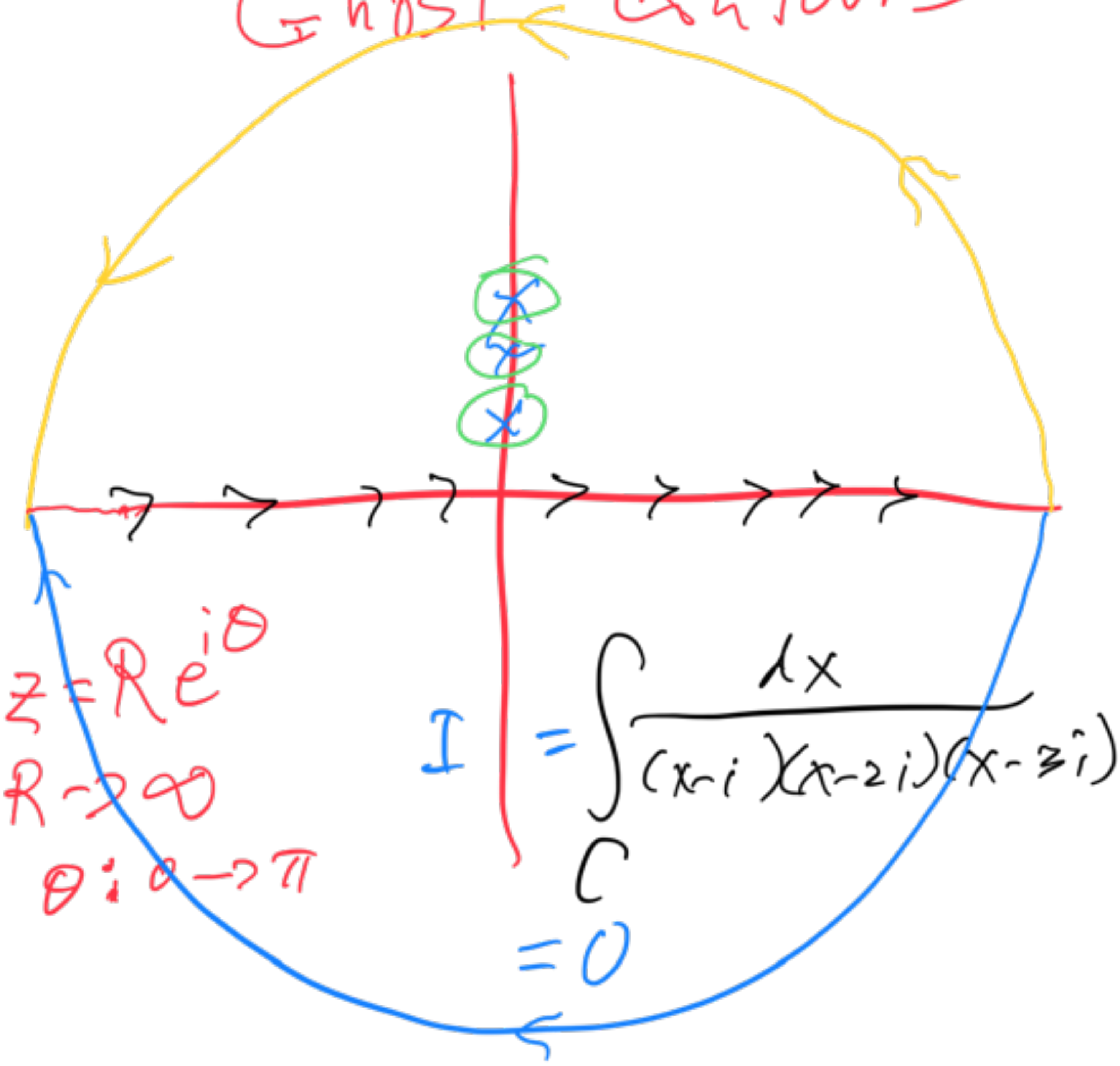
$$= \sum_{k=1}^{n} 2\pi i\, a_{-1}(z_k)$$

$$= \sum_{k=1}^{n} 2\pi i\, \mathrm{Res}(f, z_k)$$





Ghost contours



$z = Re^{i\theta}$

$R \to \infty$

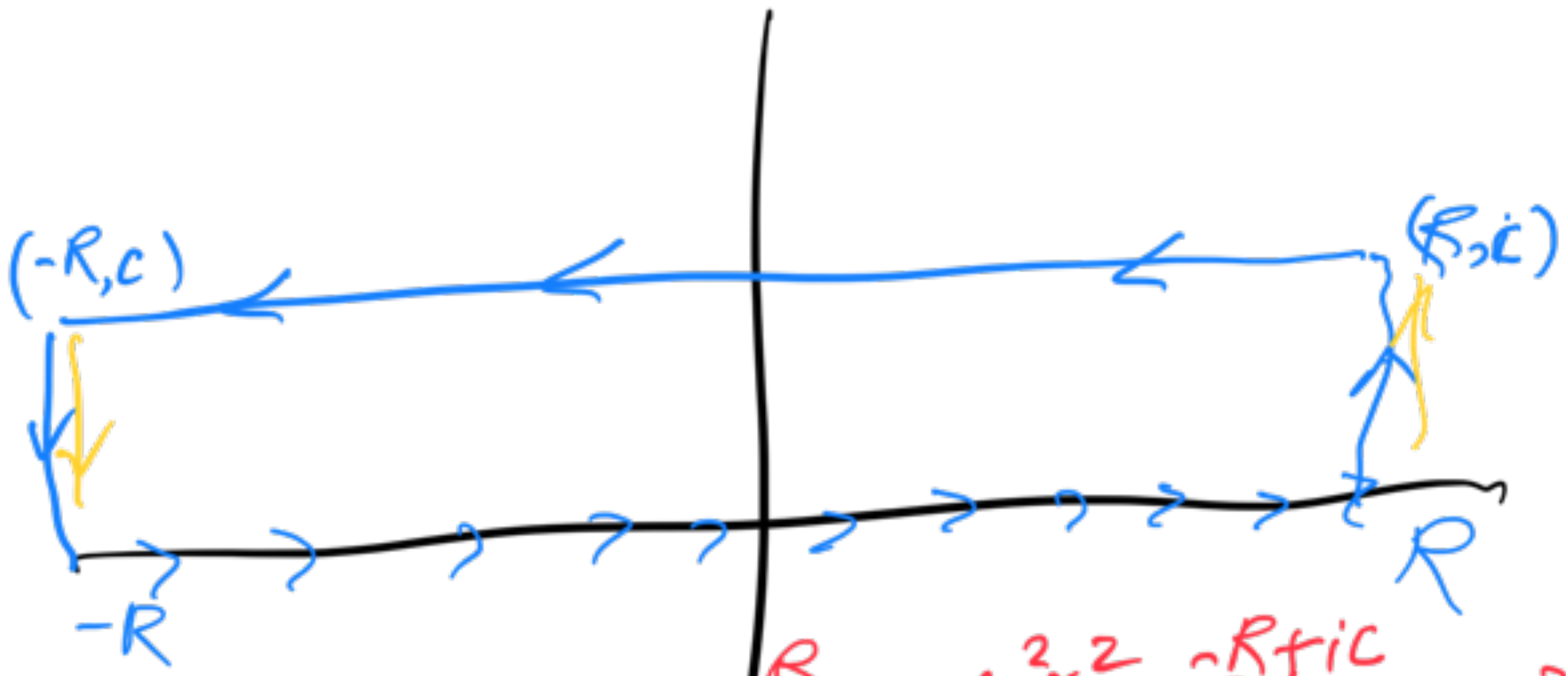
$\theta: 0 \to \pi$

$$I = \int_C \frac{dx}{(x-i)(x-2i)(x-3i)} = 0$$

$$2\pi i \left[\frac{1}{(i-2i)(i-3i)} + \frac{1}{(2i-i)(2i-3i)} + \frac{1}{(3i-i)(3i-2i)} \right] = 0$$

$$= 2\pi i \left[\frac{1}{-i(-2i)} + \frac{1}{i(-i)} + \frac{1}{2i(i)} \right]$$

$$= 2\pi i \left[-\frac{1}{2} + 1 - \frac{1}{2} \right] = 0.$$



$$\oint dz\, e^{-m^2 z^2} = \int_{-R}^{R} dx\, e^{-m^2 x^2} + \int_{R+ic}^{-R+ic} dz\, e^{-m^2 z^2}$$

$$e^{-m^2 R^2}$$

$$= 0$$

$$\int_{-R}^{R} dx\, e^{-m^2 x^2} = \int_{-R+ic}^{R+ic} dx\, e^{-m^2 (x+ic)^2}$$

$$\|$$

$$\frac{\sqrt{\pi}}{m}$$

$$I^2 = \int_{-\infty}^{\infty} dx\, e^{-m^2 x^2} \int_{-\infty}^{\infty} dy\, e^{-m^2 y^2}$$

$$= \int dx\, dy\, e^{-m^2 (x^2+y^2)}$$

$$= \int_0^{\infty} 2\pi r\, dr\, e^{-m^2 r^2}$$

$$= 2\pi \int_0^{\infty} r\, dr\, e^{-m^2 r^2} \qquad u = mr$$

$$= \frac{2\pi}{m^2}\int_0^\infty u\,du\, e^{-u^2} \qquad v = u^2$$

$$= \frac{\pi}{m^2}\int_0^\infty dv\, e^{-v} = \frac{\pi}{m^2} = I^2$$

$$I = \frac{\sqrt{\pi}}{m}$$

$$\int_{-\infty}^{\infty} dx\, e^{-m^2(x+ic)^2} = \int dx\, e^{-m^2x^2} = \frac{\sqrt{\pi}}{m}$$

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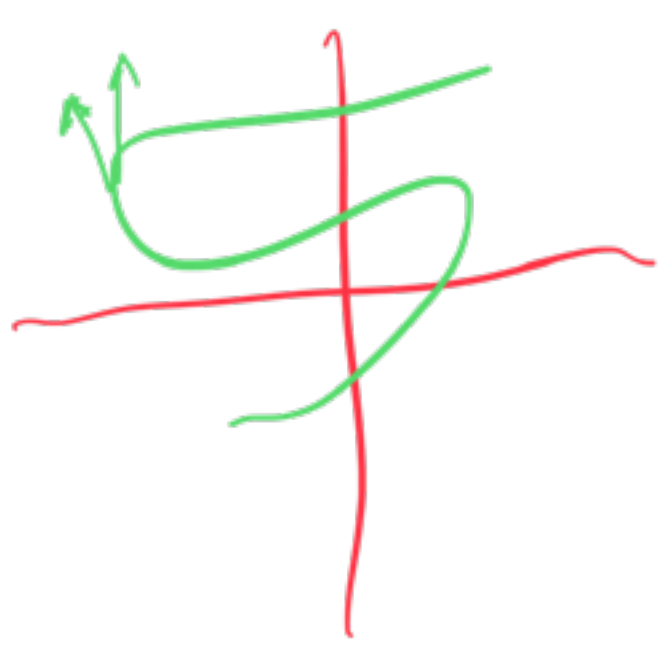
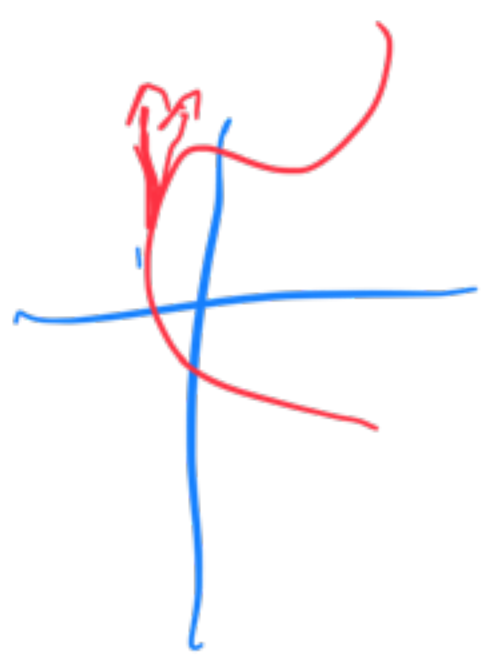




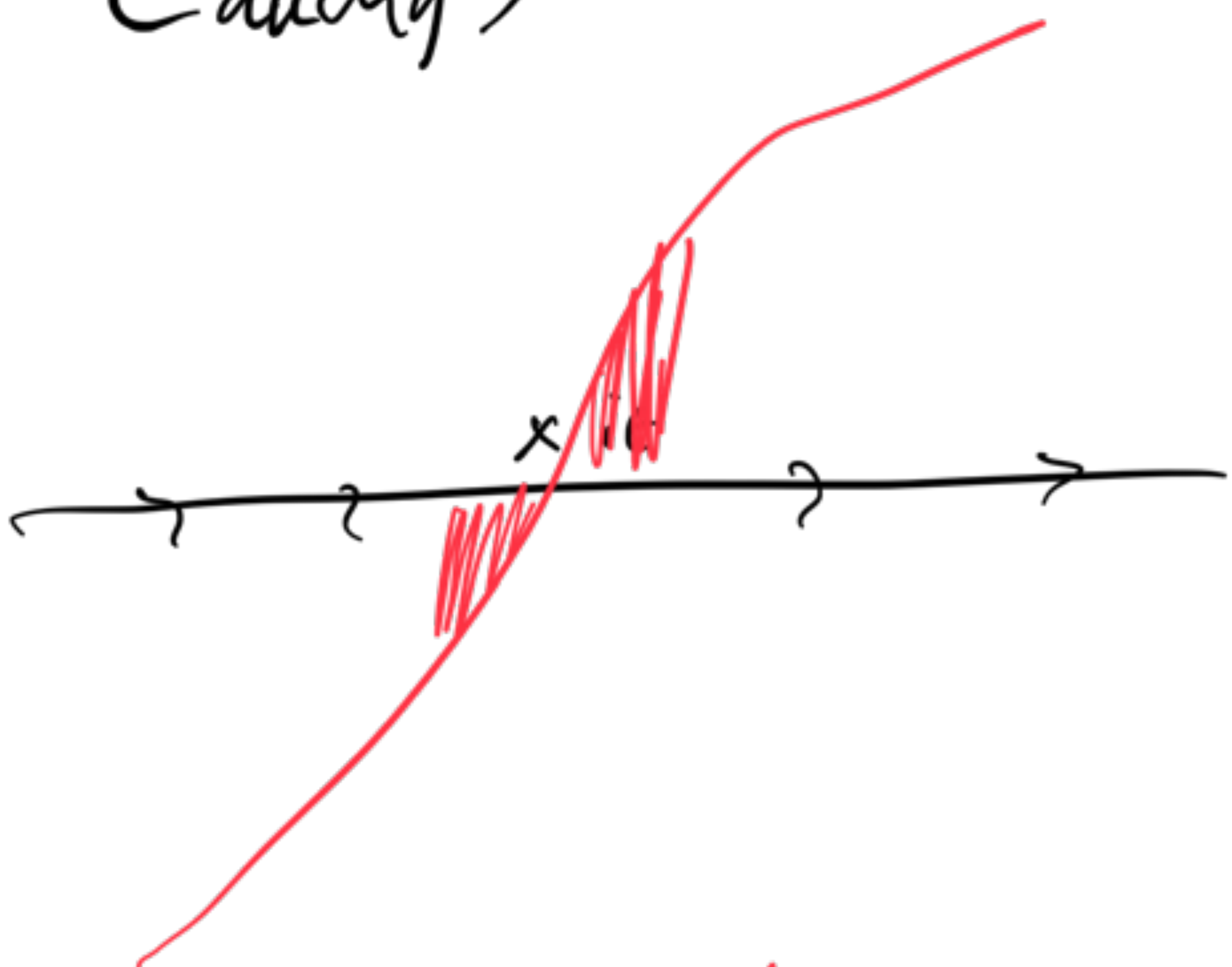
$dz' = \epsilon e^{i\theta'}$

$dz = \epsilon e^{i\theta}$

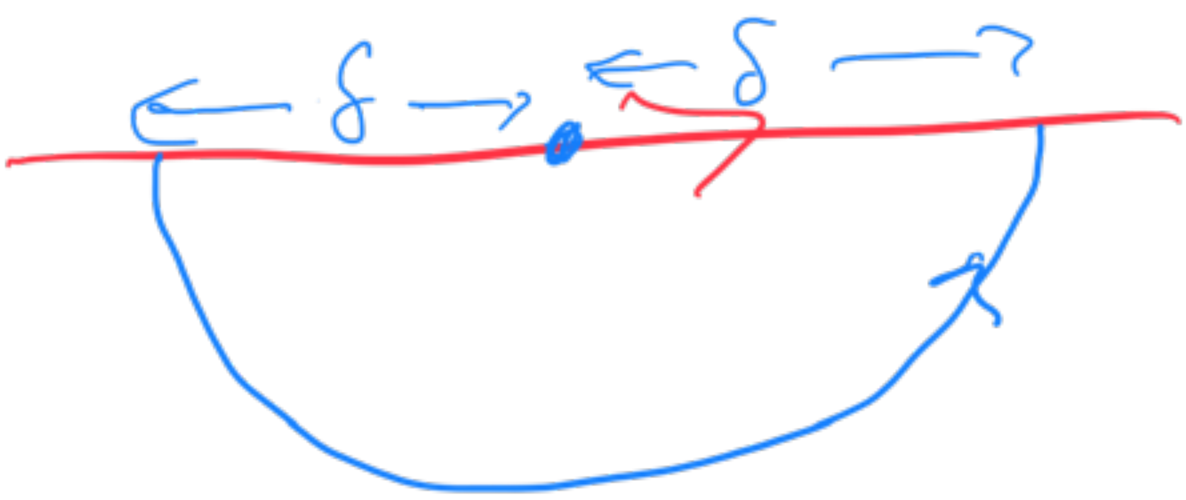
$\frac{df}{dz} = f'(z)$ independent of θ & θ' etc.

g ...

Cauchy's P.V.



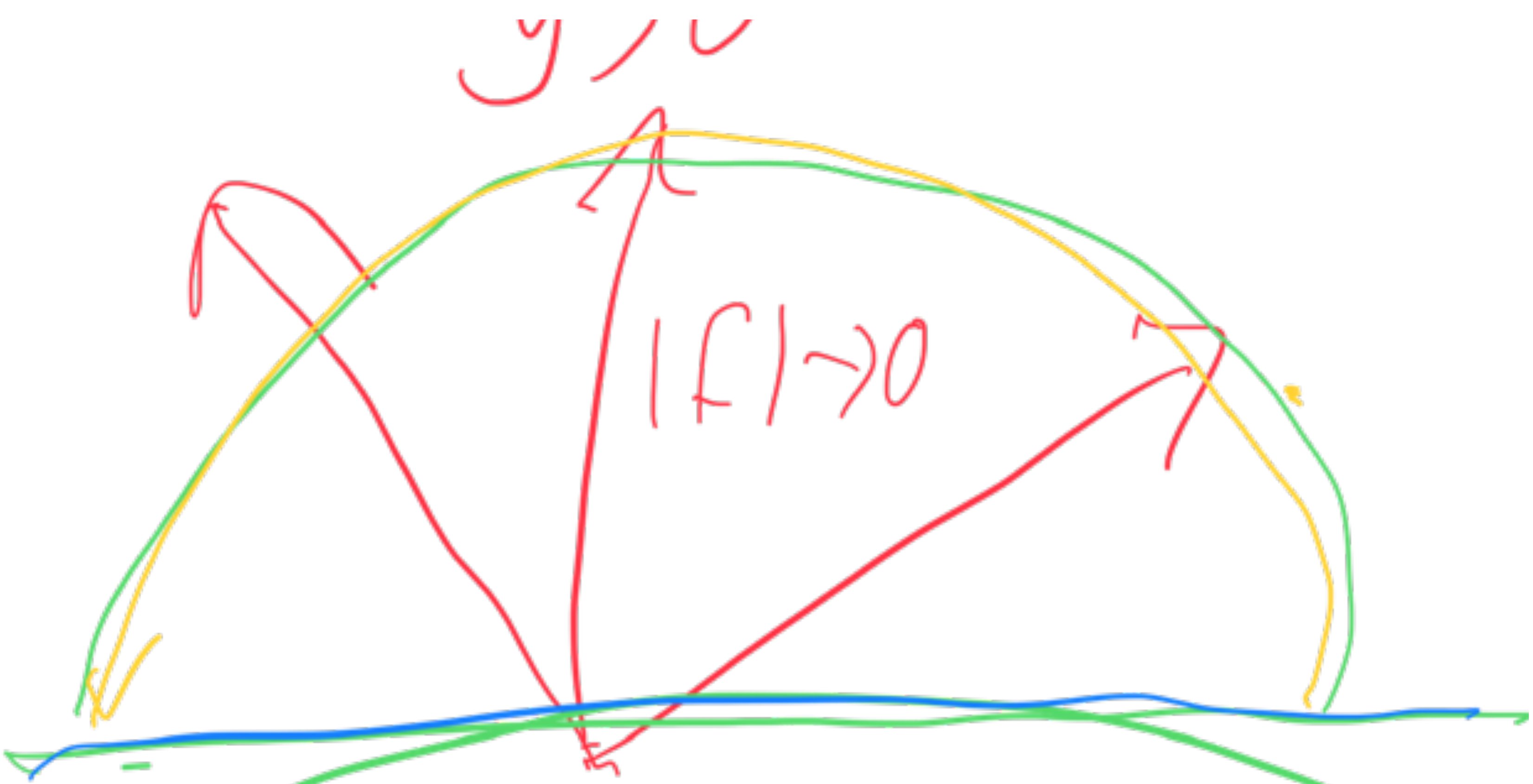
$$f(z) = \frac{1}{z}$$



$z = \delta e^{i\theta}$ $\qquad$ $\theta : \pi \to 2\pi$

$$e^{izt} = e^{i(x+iy)t} = e^{ixt} e^{-yt}$$

$y > 0$



$$f(z_0) = \frac{1}{2\pi i}\oint \frac{f(z)\,dz}{z-z_0}$$

$$f(z_0) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{f(x)\,dx}{x-z_0}$$

$$f(x_0) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} \frac{f(x)\,dx}{x-x_0-i\epsilon}$$

$$f(x_0) = \frac{1}{\pi i}\, P\int_{-\infty}^{\infty} \frac{f(x)\,dx}{x-x_0}$$

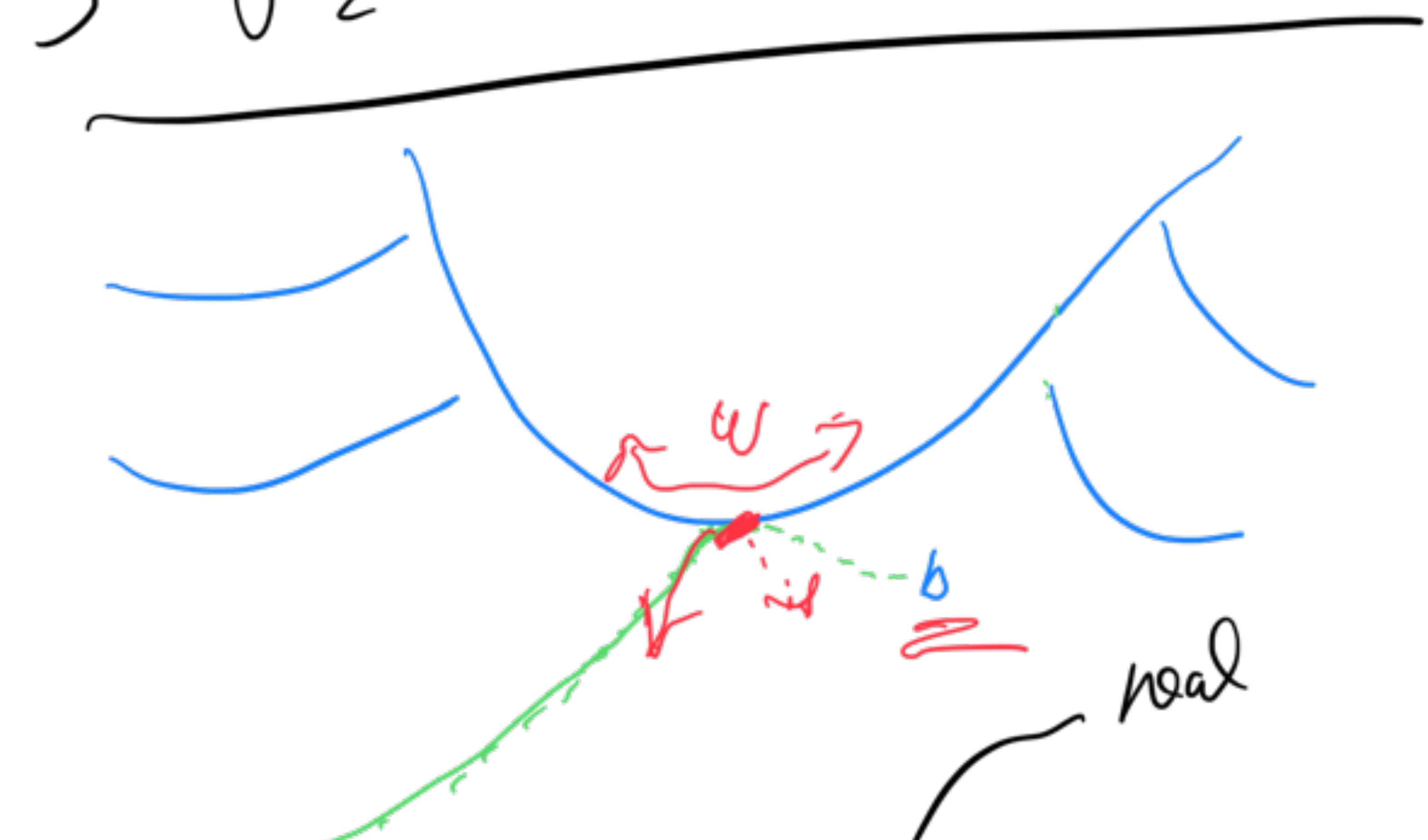
$$S = M^2 \int R \sqrt{g}\, d^4x$$

$$u(x_0) = \frac{P}{\pi} \int_0^\infty \left[\frac{v(x)}{x - x_0} + \frac{-v(x)}{-x - x_0} \right] dx$$

$$= \frac{P}{\pi} \int_0^\infty v(x)\, \frac{1}{x - x_0} + \frac{1}{x + x_0}\, dx$$

$$= \frac{P}{\pi} \int_0^\infty \sigma(x)\, \frac{2x}{x^2 - x_0^2}\, dx$$

$$S = \int \tfrac{1}{2} m v^2 - V \, dt$$

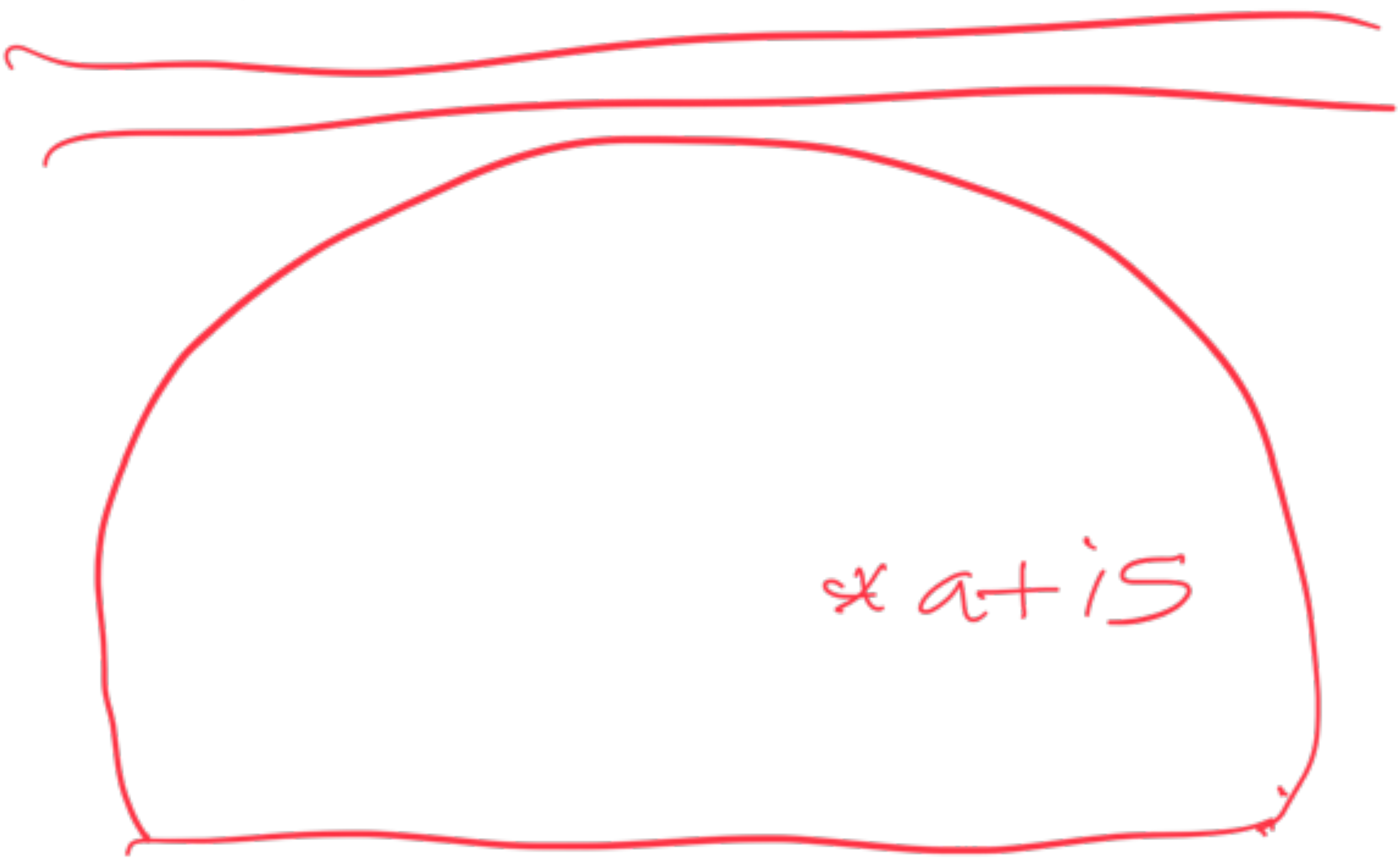


real

$$I(n) = \int_a^b dz\, h(z)\, e^{n f(z)}$$

$f = u(z) + i v(z)$

$f(z) = u(z) + i v(z)$



$$f(z) = n \log z - z$$

$$f' = \frac{n}{z} - 1 = 0$$

$$n = z = w$$

$$f'' = -\frac{n}{z^2} = -\frac{1}{n}$$

$$\sum_{j=1}^{n} a_j \frac{d^j}{dx^j} G(x-y) = \delta(x-y)$$

$$G(x-y) = \int \frac{dk}{2\pi} \frac{e^{ikx}}{\sum_{j=1}^{n} a_j (ik)^j}$$

$L f(x) = 0$ homog

$L f(x) = s(x)$ incoming

source term

$$A_i(x) = \int \frac{d^3x \, J_i(y)}{4\pi |x-y|}$$

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

nr $\frac{d}{dt}$

$$\nabla \cdot B = 0$$

$$-YX'' - XY'' = k^2 XY$$

$$-\frac{X''}{X} - \frac{Y''}{Y} = k^2$$

x y

$$-\frac{X''}{X} = a^2, \quad -\frac{Y''}{Y} = b^2$$

$$a^2 + b^2 = k^2$$

$a + b$

$$X_a(x)\, Y_b(y)$$

$$X_a(x) = e^{iax}$$

$$Y_b(y) = e^{iby}$$

$$X_a(x)\, Y_b(y) = e^{iax+iby}$$

$$= \int da\, e^{iax + i\sqrt{k^2-a^2}\, y} f(a)$$

$$\underbrace{\rho^2 \frac{P''}{P} + \rho \frac{P'}{P} + \rho^2 \lambda^2}_{P} + \underbrace{\frac{\Phi''}{\Phi}}_{\Phi} = k^2 \quad \text{cst}$$

$$(x^2 j_\ell')' + [x^2 - \ell(\ell+1)] j_\ell = 0$$

$$J_n'' + \frac{1}{z} J_n' + \left(1 - \frac{n^2}{z^2}\right) J_n = 0$$

$$\left(z^2 J_n'' + z J_n'\right) + \left((z^2 - n^2) J_n\right) + 0$$

$$\left(z^2 J_n'\right)' = 2z J_n' + z^2 J_n''$$

x^i is what you think it is

$x^1 = x \qquad x^2 = y \qquad x^3 = z$

$x^0 = ct$

$x_1 = x \qquad x_2 = y \qquad x_3 = z$

$x_0 = -ct$

$$\frac{\partial}{\partial x^1} = \partial_1 \qquad \frac{\partial}{\partial x^2} = \partial_2 = \partial_y$$

$$\partial_0 = \frac{\partial}{\partial ct} = \frac{\partial}{\partial x^0}$$

$$\partial^1 = \partial_1 \quad \partial^2 = \partial_2 \quad \partial^3 = \partial_3$$

$$\partial^0 = -\partial_0$$

$$\eta = \begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

$$\eta' = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\underline{p'^2} = p^{0\,2} - \vec{p}^{\,2} = E^2 - \vec{p}^{\,2} = \underline{m^2}$$

$$p^2 = -m^2$$

$$[q_j, p_k] = i\hbar\,\delta_{jk}$$

$$[q_j, q_k] = 0 = [p_j, p_k]$$

$$\dot{\phi} = \pi$$

$$(\not{\partial} + m)\chi = 0$$

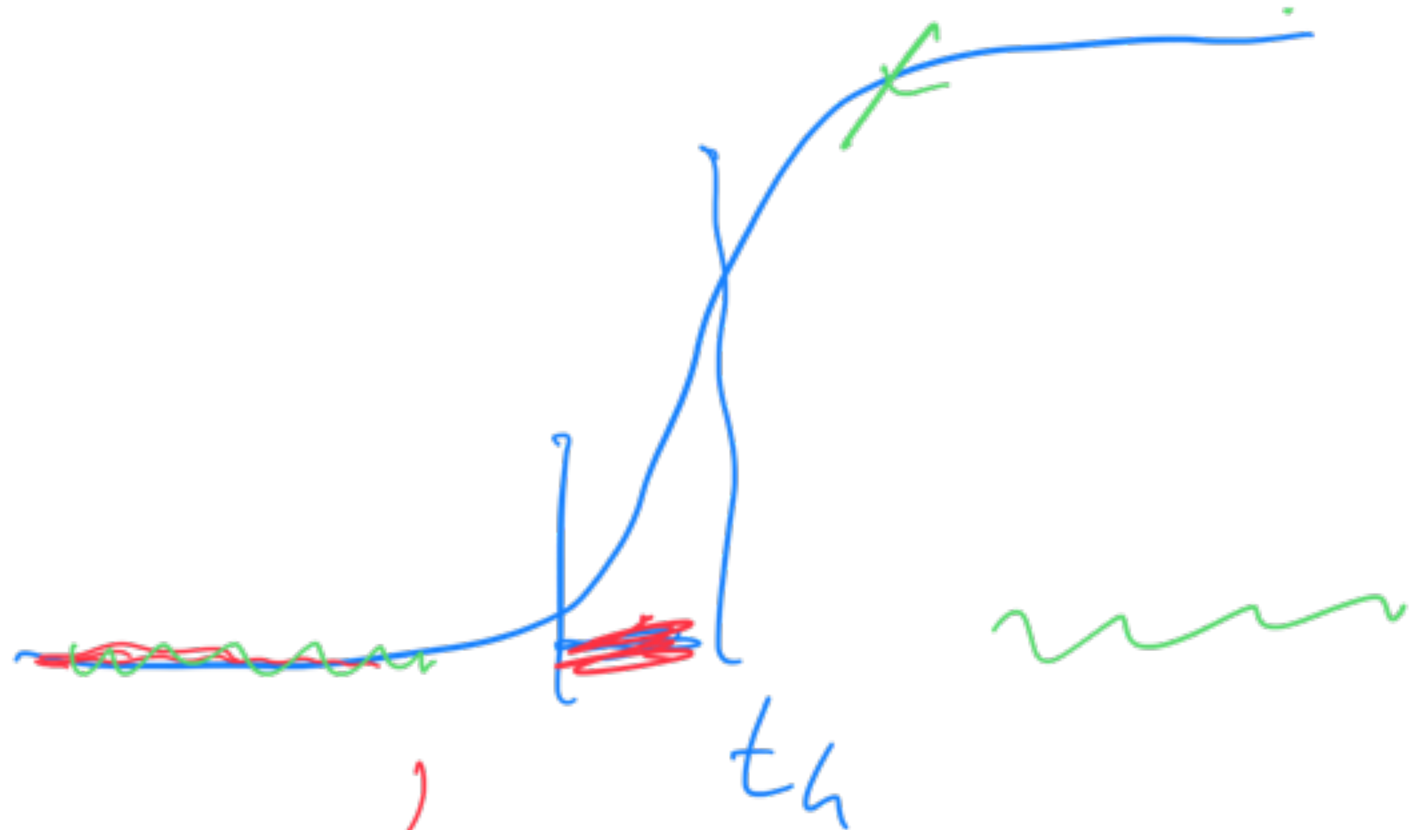
$$x^2\,dx + y\,dy = 0$$

$$0 = \int_0^x x^2\,dx + \int_0^y y\,dy$$

$$0 = \frac{x^3}{3} + \frac{y^2}{2}$$

$$\frac{y^2}{2} = -\frac{x^3}{3}$$

$$y = \sqrt{-\frac{2}{3}x^3}$$



$$= \frac{1}{a}$$

$$\gamma^0 = -i\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\vec{\gamma} = -i\begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$$