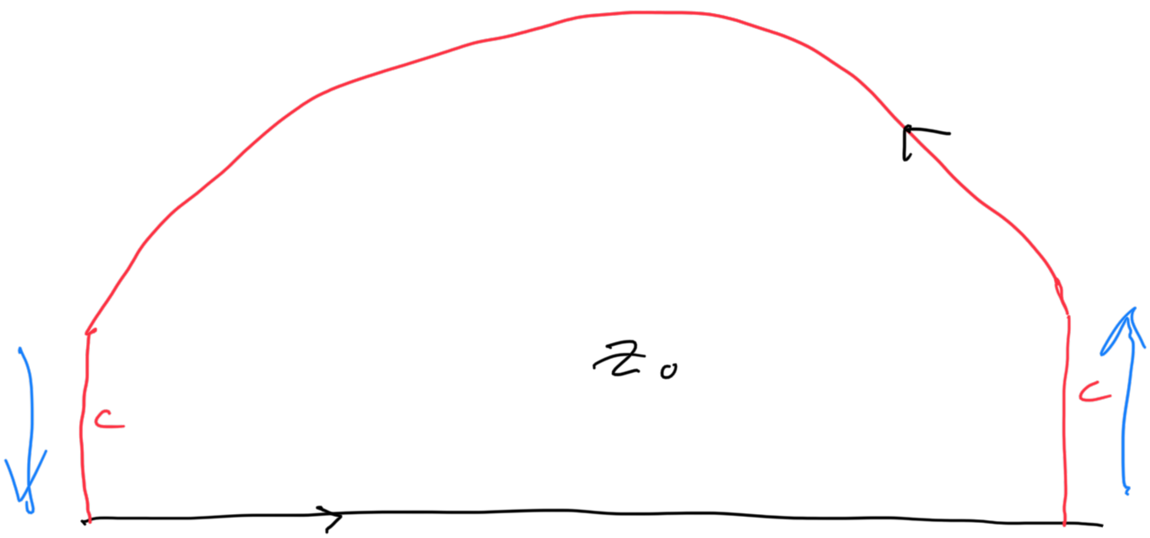
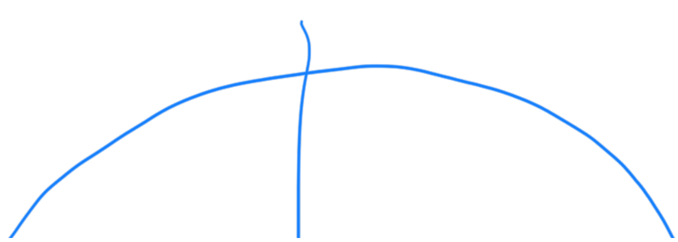
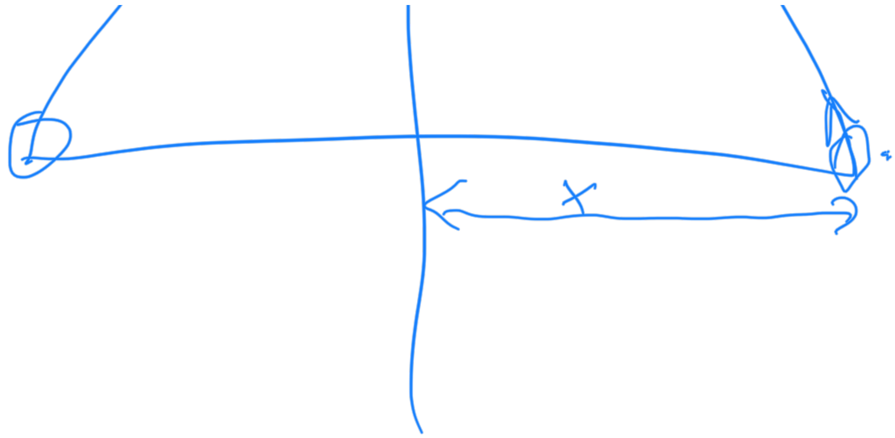


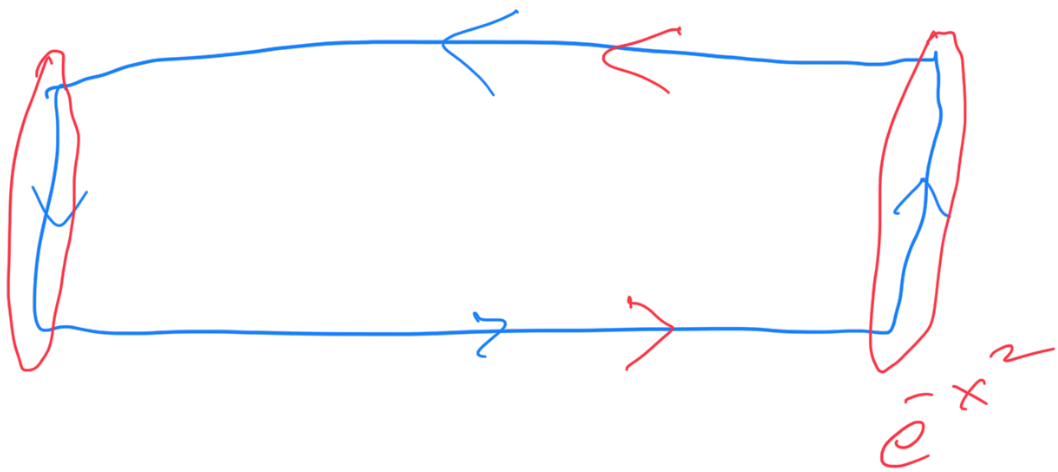
$$\left| \int_0^c dy \, \frac{f(r+iy)}{r+iy-z_0} \right| \lesssim \frac{|f(r+iy)|}{r}$$

$$\frac{1}{G} \int R \sqrt{g} \, d^4x$$

$$\longrightarrow \left| f(x+iy) \right| \to 0 \text{ as } y \to \infty$$







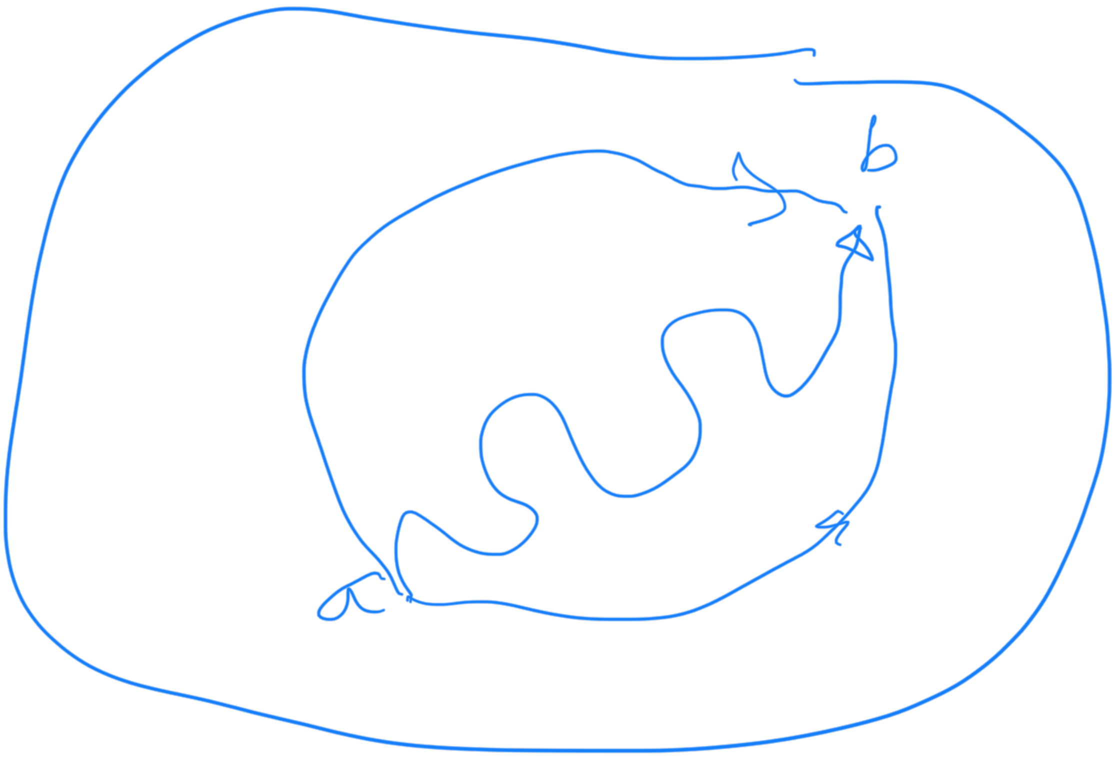
$$A(x,t) = \int e^{i(p\cdot x\,c - Et)/\hbar} A(p)\, d^3p$$

$$\left(\frac{\partial^2}{\partial t^2} - \nabla\cdot\nabla + \frac{m^2}{\hbar^2}\right)A = \int\left(-\frac{E^2}{\hbar^2} + \frac{\vec{p}^{\,2}c^2}{\hbar^2} + m^2\right) A(p)\, d^3p\, e^{i(p\cdot x - Et)/\hbar}$$

$$= 0$$

$$E^2 = c^2 \vec{p}^{\,2} + m^2 c^4$$

$$I(r) = \int_a^b dz \; h(z)\, e^{r f(z)}$$

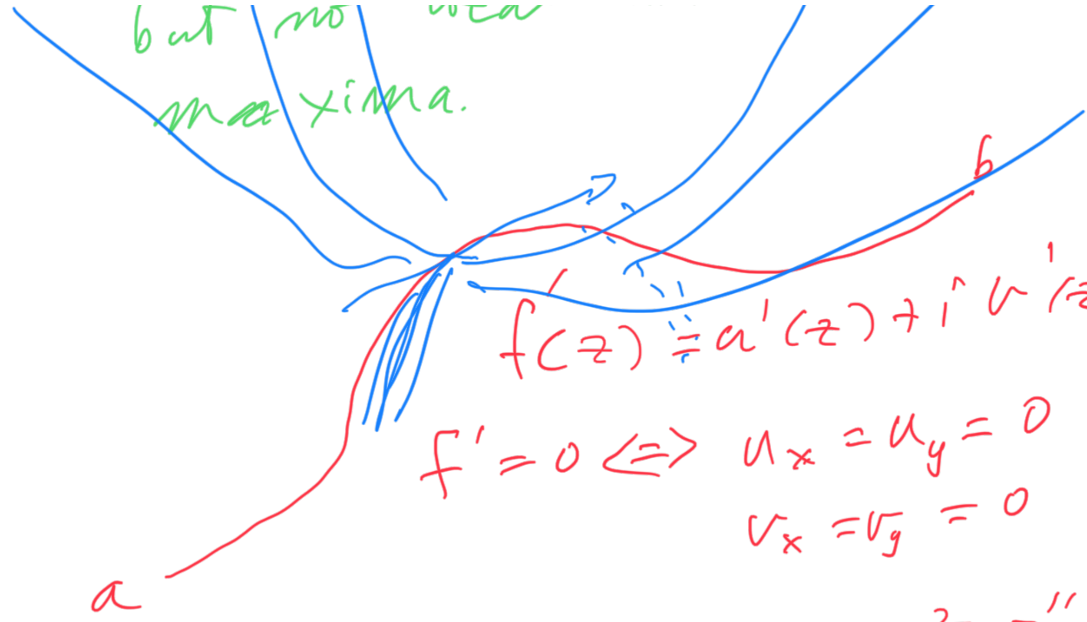


$e^{r(u(z)+iv(z))}$ sensitive to $u(z)$

u and v have saddle points

local minima or

but no ... maxima.



$f(z) = u'(z) + i\,v'(z)$

$f' = 0 \iff u_x = u_y = 0$

$v_x = v_y = 0$

$$f(z) = f(w) + \frac{1}{2}(z-w)^2 f''(w)$$

w is fixed at s.p.

$f''(w) = \rho e^{i\phi}$

$$(-1)\frac{dy^{-1}}{dy} = (-1)(-1)\,y^{-2}\Big|_{y=1} = 1$$

$$= -2\,y^{-3}\Big|_{y=1} \to \not{2} = 2!$$

$$L f(x) = \begin{cases} 0 & \text{homogeneous} \\ s(x) & \text{source inhom} \end{cases}$$

$$L(a_1 f_1 + a_2 f_2) = a_1 L f_1 + a_2 L f_2$$

$$L = \sum c_m \frac{d^m}{dx^m}$$

$$f(x) = \int dk\, e^{ikx}$$

$$Lf = \int dk \sum_{n=0}^{n} c_m (ik)^m e^{ikx} = 0$$

$$f(x) = \int dk\, \delta\left(\sum c_m (ik)^m\right) h(k)\, e^{ikx}$$

$$L f(x) = \int dk \left(\sum c_m (ik)^m\right) \delta\left(\sum c_m (ik)^m\right) e^{ikx} h(k)$$

$$= 0$$

$$L f(x) = s(x)$$

$$\int dk \left(\sum c_m (ik)^m\right) \tilde{f}(k) e^{ikx}$$

$$\int dk\, e^{ikx}\, \tilde{s}(k)$$

$$= \int dk\, e$$

$$\tilde{f}(k) = \frac{\tilde{s}(k)}{\sum c_m (ik)^m} \leftarrow \text{real or imag}$$

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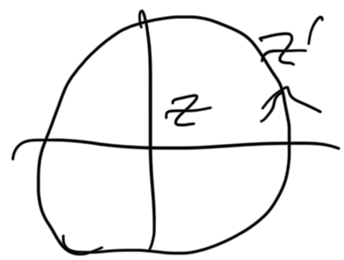
$$G_1(x,z) = \sum_{n=0}^{\infty} J_n(x) z^n$$



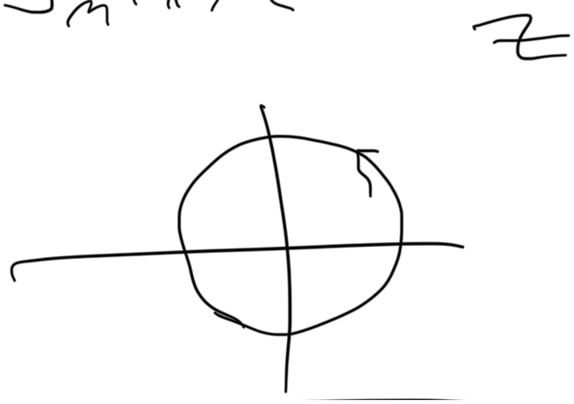
$$= \frac{1}{2\pi i} \oint_C \sum_n e^{\frac{x}{2}\left(z' - \frac{1}{z'}\right)} \frac{dz'}{z'^{n+1}} z^n$$

$$= \frac{1}{2\pi i} \oint \frac{dz'}{z'} e^{\frac{x}{2}\left(z' - \frac{1}{z'}\right)} \sum_{n=0}^{\infty} \left(\frac{z}{z'}\right)^n$$

$$= \frac{1}{2\pi i} \oint \frac{dz'}{z'} e^{\frac{x}{2}\left(z' - \frac{1}{z'}\right)} \frac{1}{1 - \frac{z}{z'}}$$

$$G_2(x,z) = \sum_{n=-\infty}^{-1} J_n(x) z^n$$

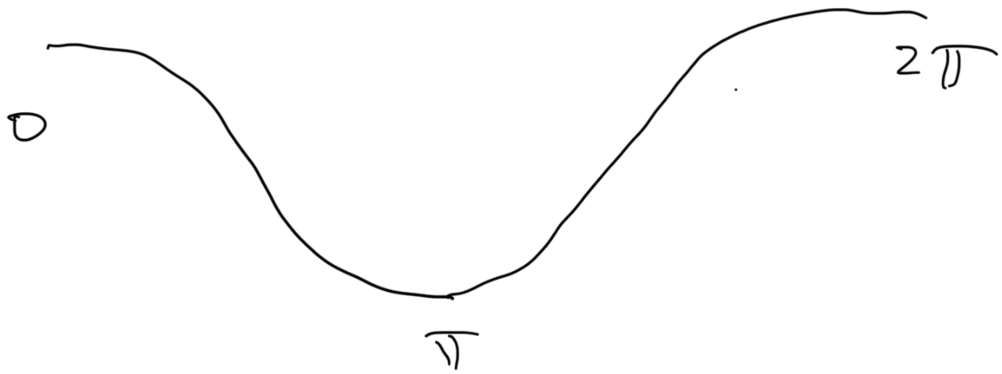
$$G = G_1 + G_2$$



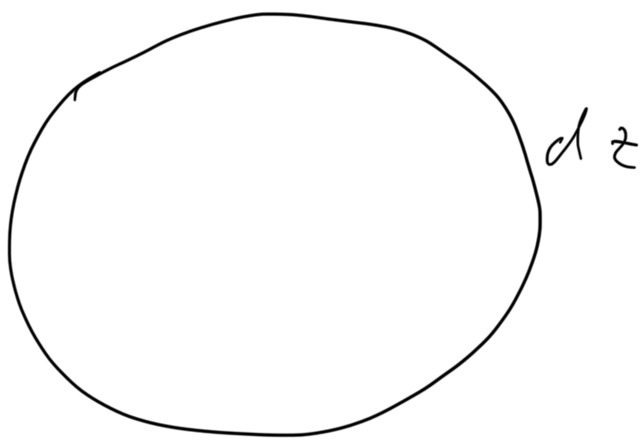
$z = e^{i\theta}$

$$I_a = \frac{1}{2}\int_0^{2\pi} \frac{d\theta}{a+\cos\theta} = \frac{1}{2}\int_0^{2\pi} \frac{dz}{iz\left(a+\frac{z+z^*}{2}\right)}$$

quadratic



$dz = i e^{i\theta} d\theta$ $\qquad d\theta = \frac{dz}{ie^{i\theta}} = \frac{dz}{iz}$



$$I = \int_{-\infty}^{\infty} \frac{dx}{1+x^6}$$

$x^6 = -1 = e^{i\pi}$

$x = e^{i\pi/6}$

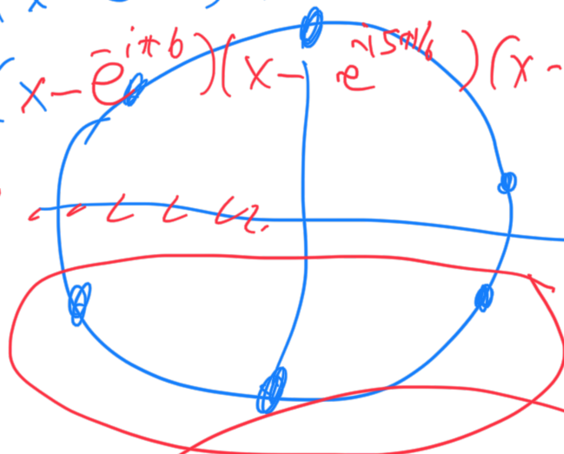
$$= \int \frac{dx}{(x-e^{i\pi/6})(x-e^{5i\pi/6})(x-e^{i\pi/2})(x-e^{-i\pi/6})(x-e^{-i5\pi/6})(x-e^{-i\pi/2})}$$

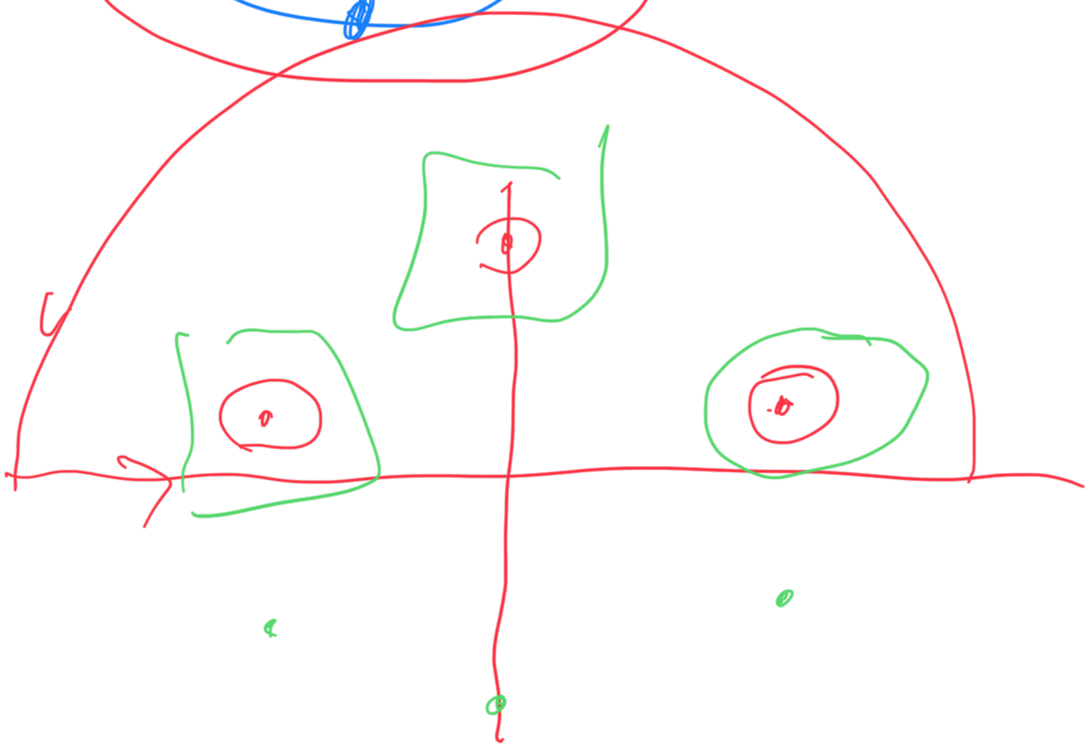
$e^{-i\pi/6}$

$e^{5i\pi/6}$ $e^{-5i\pi/6}$

$e^{3i\pi/6}$, $e^{-i\pi}$

$e^{i\pi - \frac{i\pi}{6}}$ $e^{i\pi/2}$ $e^{-i\pi/\ldots}$





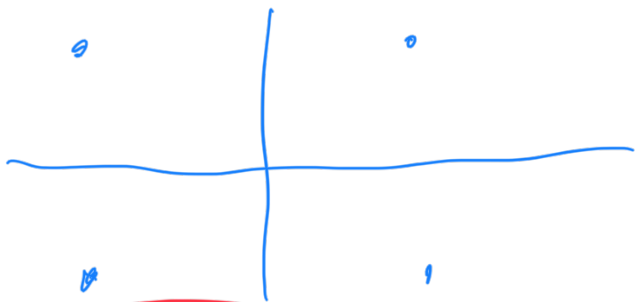
$$= 2\pi i \frac{1}{(e^{i\pi/6}-e^{5i\pi/6})(e^{i\pi/6}-e^{i\pi/2})(e^{i\pi/6}-e^{-i\pi/6})(e^{i\pi/6}-e^{-i5\pi/6})(e^{i\pi/6}-e^{-i\pi/2})}$$

x^4 $e^{i\pi/4}$ $e^{-i\pi/4}$

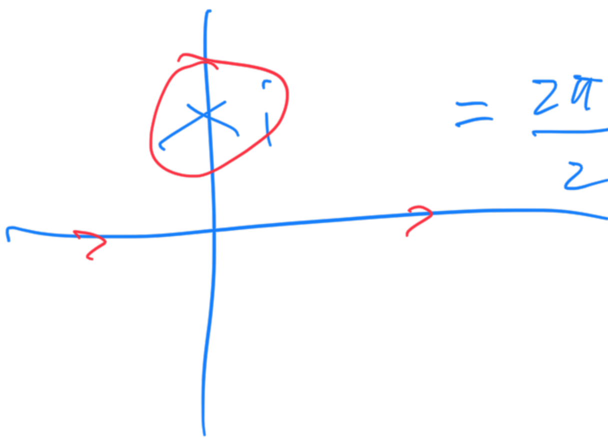
$x' = -1$

$e^{i3\pi/4}$

$e^{-i3\pi/4}$



$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \oint \frac{dz}{(z-i)(z+i)}$$



$$= \frac{2\pi i}{2i} = \pi$$

Thursday 8 October 2020

$$\phi = X(x)\, Y(y)\, Z(z)$$

$$\frac{\partial^2}{\partial x^2}\left(X(x)\,Y(y)\,Z(z)\right) = Y(y)\,Z(z)\,\frac{\partial^2 X(x)}{\partial x^2}$$

$$X'' + XZY'' + XYZ'' = 0$$

$$\Delta\phi = \nabla\cdot\nabla\phi = YZX'' + XZY'' + XYZ'' = 0$$

$$\frac{\Delta\phi}{XYZ} = \frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = 0$$

$$\begin{array}{ccc} x & y & z \\ \shortparallel & \shortparallel & \shortparallel \\ c'' & c' & c \end{array} \qquad c + c' + c'' = c$$

$$X'' = c''X \qquad X = e^{x\sqrt{c''}}$$

$$X' = \sqrt{c''}\,X \qquad X'' = c''X$$

$$X \sim e^{x} \qquad Y \sim e^{y} \qquad Z \sim e^{z}$$

$$X \sim e^{ix} \qquad Y \sim e^{iy} \qquad Z \sim e^{z}$$

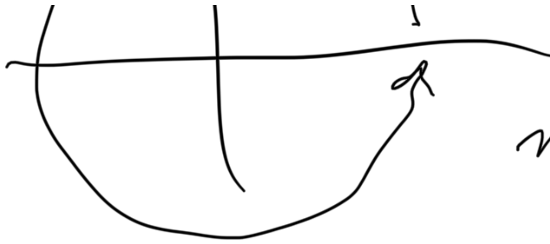
$$-\frac{1}{X}X'' - \frac{Y''}{Y} = k^2$$

$$\begin{array}{ccccc} \shortparallel & & \shortparallel & & \\ a^2 & + & b^2 & = & h^2 \end{array}$$

$$X = e^{x\sqrt{a}} \qquad Y = e^{y\sqrt{b}}$$

$L\phi$

$e^{im\phi}$



e

n is an integer

$$e^{i2\pi n} = 1 = e^{in\theta}\Big|_0 = e^0$$

in a wedge

$$\rho^2 P''_{km}(\rho) = \rho^2 \frac{d^2}{d\rho^2} J_m(k\rho) \qquad x = k\rho$$

$$= k^2\rho^2 \frac{1}{k^2} \frac{d^2}{d\rho^2} J_m(k\rho)$$

$$= x^2 \frac{d^2}{dx^2} J_m(x)$$

$$-\underline{X''} - \underline{Y''} - \underline{Z''} = k^2$$

$X \quad Y \quad Z \quad 4$

$\underbrace{x \quad y \quad z \quad \text{constant}}$

$$-X'' = a^2 X$$

$X = e^{iaX}$

$X' = ia\, e^{iaX}$

$X'' = -a^2 e^{iaX} = -a^2 X$

$$0 = a^{+}(p,s)\, a^{+}(p,s)\, |\psi\rangle$$

$$= a^{+}(p,s)\, |p,s;\psi\rangle$$

$(7.59) \Rightarrow \quad y = g(x, x_0, y_0)$

$h = 1, \quad x = \dfrac{a}{n+c}$ Zipf's law

$x = \dfrac{a}{\cancel{n}} = \dfrac{a}{n}$

$x \quad \dfrac{x}{2} \quad \dfrac{x}{3} \quad \dfrac{x}{4}$

$n = a \quad ,$ $\quad x = \dfrac{a}{}$

'' $\bar{x}$ because '' n

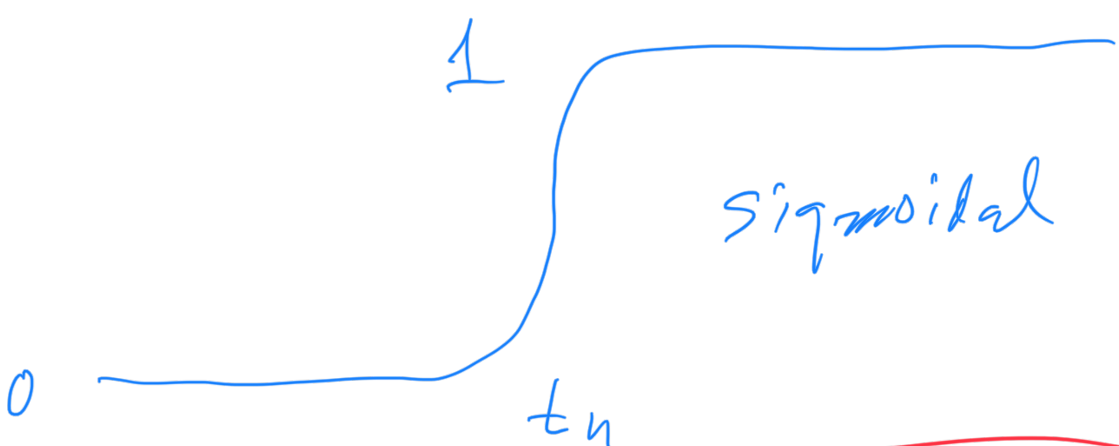
$$\frac{df}{dt} = af(1-f)$$

$$\frac{df}{f(1-f)} = a\,dt$$

$$\int \frac{df}{f} + \int \frac{df}{1-f} = a(t - t_n)$$

$$\log f - \log 1-f = \log \frac{f}{1-f} = a(t-t_n)$$

$$f = \frac{e^{a(t-t_n)}}{1+e^{a(t-t_n)}}$$



$$\int_{x_0}^{x} \frac{U(x')}{R(x')}\,dx' + \int_{y_0}^{y} \frac{S(y')}{V(y')}\,dy' = 0$$

$$F(x, x_0) + G(y, y_0) = 0$$

$$y = g(x, x_0, y_0)$$

$$P\,dx + Q\,dy = \frac{\partial \phi}{\partial x}\,dx + \frac{\partial \phi}{\partial y}\,dy$$

EXACT

$$P = \frac{\partial \phi}{\partial x} \qquad Q = \frac{\partial \phi}{\partial y}$$

$$\frac{\partial P}{\partial y} = \frac{\partial^2 \phi}{\partial x \partial y} = \frac{\partial Q}{\partial x}$$

integrability

$$V = \frac{1}{r} = \frac{1}{\sqrt{x_1^2 + x_2^2 + x_3^2}}$$

$$V(t\vec{x}) = \frac{1}{\ldots} = \frac{1}{t} V(x$$

$$\sqrt{t^2 (\dot{x}^2)} \qquad t$$

$$= t^{-1} V(x)$$

homog degree -1.

$$\langle T \rangle = -\frac{1}{2} \left\langle \frac{1}{r} \right\rangle.$$

$$V(r) = \sum m_k \omega_k^2 r_k^2$$

$$V(tr) = t^2 V(r)$$

homog. deg. 2

$$\langle T \rangle = \langle V \rangle.$$