$$(AB)^\dagger_{ik} = (AB)^*_{ki} = (A^* B^*)_{ki} = \sum_\ell A^*_{k\ell} B^*_{\ell i}$$

$$= \sum_\ell A^\dagger_{\ell k} B^\dagger_{i\ell} = \sum_\ell B^\dagger_{i\ell} A^\dagger_{\ell k} = (B^\dagger A^\dagger)_{ik}$$

So $$(AB)^\dagger = B^\dagger A^\dagger$$

$$0 \le (f-\lambda g, f-\lambda g) = \langle f-\lambda g | f-\lambda g\rangle$$ for all f, g in space

$$0 \le \langle f|f\rangle - \lambda^* \langle g|f\rangle - \lambda \langle f|g\rangle + |\lambda|^2 \langle g|g\rangle$$

Set $\lambda = x + iy$. Then $0 \le \langle f|f\rangle - x(\langle g|f\rangle + \langle f|g\rangle) + (x^2+y^2)\langle g|g\rangle$
$- iy(\langle f|g\rangle - \langle g|f\rangle) \equiv S$

So $0 = \frac{\partial S}{\partial x} = \frac{\partial S}{\partial y}$. So $2x\langle g|g\rangle = \langle f|g\rangle + \langle g|f\rangle$

$2y\langle g|g\rangle = i(\langle f|g\rangle - \langle g|f\rangle)$.

So $\langle g|g\rangle = 0 \Rightarrow \langle f|g\rangle = \langle g|f\rangle = 0$ for all f in space.

$$[k \times (k \times V)]_i = \sum_{j,\ell} \epsilon_{ij\ell} k_j (k\times V)_\ell$$

$$= \sum_{j,\ell,m,n} \epsilon_{ij\ell} k_j \epsilon_{\ell mn} k_m V_n = \sum_{j,m,n} (\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}) k_j k_m V_n$$

$$= k_i k\cdot V - k^2 V_i$$ So $$\vec V = \frac{1}{\vec k^2}\left[\vec k (k\cdot V) - k\times(k\times V)\right]$$

$$\vec A(x) = \int d^3k\, e^{ik\cdot x} \hat A(k)$$

$$-\nabla^2 \vec A(x) = -\int d^3k\, i\vec k \cdot i\vec k\, e^{ik\cdot x} \hat A(k) = \mu_0 \vec J(x)$$

$$= \int d^3k\, \vec k^2 e^{ik\cdot x} \tilde A(k) = \mu_0 \int d^3k\, e^{ik\cdot x} \tilde J(k)$$

So $\tilde{A}(k) = \frac{\mu_0}{\vec{k}^2} J(k)$.

So $\hat{A}(x) = \int d^3k \, \frac{e^{ik\cdot x}\mu_0 \tilde{J}(k)}{k^2} = \int d^3k \frac{\mu_0}{k^2} e^{ik\cdot x} \int \frac{d^3y}{(2\pi)^3} e^{-ik\cdot y} J(y)$

$$= \mu_0 \int d^3y \, G(x-y) J(y) = \mu_0 \int d^3y \frac{J(y)}{4\pi|\vec{x}-\vec{y}|}$$

$$D(\partial)\, G(x-y) = \delta(x-y)$$

$$D(\partial)\, G(x) = \delta(x)$$

$$D(\partial) \int \tilde{G}(k) \frac{dk}{2\pi} e^{ikx} = \int e^{ikx} \frac{dk}{2\pi}$$

$$\int D(ik)\tilde{G}(k) e^{ikx} \frac{dk}{2\pi} = \int e^{ikx} \frac{dk}{2\pi}$$

$$D(ik)\tilde{G}(k) = 1$$

$$\tilde{G}(k) = \frac{1}{D(ik)}$$

$$D(\partial) = -\nabla\cdot\nabla = -\partial\cdot\partial$$

$$\tilde{G}(\vec{k}) = \frac{1}{D(i\vec{k})} = \frac{1}{-(ik)^2} = \frac{1}{\vec{k}^2}$$

$$G(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \frac{e^{i\vec{k}\cdot\vec{x}}}{\vec{k}^2} = \frac{1}{4\pi|\vec{x}|}$$

$$D(\partial) = \partial^4 - \partial^2 + m^2$$

$$\quad (ik)^4 \quad (ik)^2 \quad m^2$$

$$D(ik) = (ik) - (ik) + m$$

$$= k^4 + k^2 + m^2$$

$$G(x) = \int \frac{dk}{2\pi} \frac{e^{ik\cdot x}}{k^4 + k^2 + m^2}$$

$$\left(\partial^4 - \partial^2 + m^2\right) f(x) = j(x)$$

$$\left(\partial^4 - \partial^2 + m^2\right) \int \frac{e^{ikx}}{2\pi} \tilde{f}(k)\, dk = \int \tilde{j}(k)\, e^{ikx} \frac{dk}{2\pi}$$

$$\int \frac{e^{ikx}}{2\pi} \left(k^4 + k^2 + m^2\right) \tilde{f}(k)\, dk =$$

$$\tilde{f}(k) = \frac{\tilde{j}(k)}{k^4 + k^2 + m^2}$$

$$f(x) = \int \tilde{f}(k)\, e^{-ikx}\, dk = \int \frac{\tilde{j}(k)}{k^4 + k^2 + m^2} e^{-ikx}\, dk$$

$$\neq 0$$