$$\binom{n-1}{k} + \binom{n-1}{k-1} =$$

$$\frac{(n-1)!}{k!\,(n-k-1)!} + \frac{(n-1)!}{(k-1)!\,(n-k)!}$$

$$= \frac{(n-1)!}{(k-1)!\,(n-k-1)!}\,\frac{1}{k} + \frac{(n-1)!}{(k-1)!\,(n-k-1)!}\,\frac{1}{n-k}$$

$$= \left(\frac{1}{k} + \frac{1}{n-k}\right)\frac{(n-1)!}{(k-1)!\,(n-k-1)!}$$

$$= \frac{n-k+k}{k\,(n-k)}\left(\frac{(n-1)!}{(k-1)!\,(n-k-1)!}\right]$$

$$= \frac{n!}{k!\,(n-k)!} = \binom{n}{k}$$

$$[fg]^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f^{(k)} g^{(n-k)}$$

$$[fg]^{(n-1)} = \sum_{k=0}^{n-1} \binom{n-1}{k} f^{(k)} g^{(n-1-k)}$$

$$[fg]^{(n)} = \sum_{k=0}^{n-1} \binom{n-1}{k} \left[f^{(k+1)} g^{(n-1-k)} + f^{(k)} g^{(n-k)} \right]$$

$$= \sum_{k=0}^{n-1} \binom{n-1}{k} f^{(k+1)} g^{n-(k+1)} + \sum_{k=0}^{n-1} f^{(k)} g^{(n-k)} \binom{n-1}{k}$$

$$\binom{n-1}{n} = 0$$

$$= \sum_{k=0}^{n-1} \binom{n-1}{k} f^{(k+1)} g^{n-(k+1)} + \sum_{k=0}^{n} f^{(k)} g^{(n-k)} \binom{n-1}{k}$$

$j = k+1$ $\quad$ $k = j-1$

$$= \sum_{j=1}^{n} \binom{n-1}{j-1} f^{(j)} g^{(n-j)} + \sum_{j=0}^{n} f^{(j)} g^{(n-j)} \binom{n-1}{j}$$

$$\binom{n-1}{-1} = 0$$

$$= \sum_{j=0}^{n} \left[\binom{n-1}{j-1} + \binom{n-1}{j} \right] f^{(j)} g^{(n-j)}$$

$$= \sum_{j=0}^{m} \binom{m}{j} f^{(j)} g^{(m-j)}$$

$|x| \ll \pi$

$$q(x) \simeq Np \sum_{k=1}^{\infty} \frac{2^{2k} B_{2k}}{(2k)!} x^{2k-1}$$

$$= Np\left(\frac{2^2 B_2}{2} x + \frac{2^4 B_4 x^3}{4!} + \frac{2^6 B_6 x^5}{6!} + \cdots \right)$$

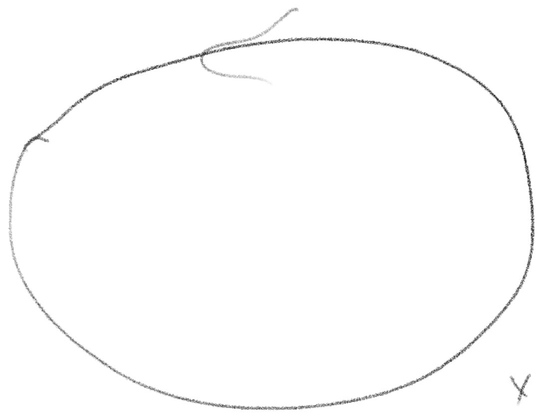
$$= Np\left(2 B_2 x + \frac{2 B_4}{3} x^3 + \frac{4 B_6}{45} x^5 \right)$$

$$= Np\left(\frac{x}{3} - \frac{x^3}{45} + \frac{2x^5}{945} + \cdots \right)$$

$$G(x,z) = \sum_{n=-\infty} J_n(x) z$$

$$= \sum_{n=-\infty}^{\infty} z^n \frac{1}{2\pi i} \oint \frac{e^{\frac{x}{2}\left(z' - \frac{1}{z'}\right)}}{z'^{\,n+1}} \, dz'$$

$$= \frac{1}{2\pi i} \oint \sum_{n=-\infty}^{\infty} \frac{z^n}{z'^{\,n+1}} e^{\frac{x}{2}\left(z' - \frac{1}{z'}\right)} \, dz'$$

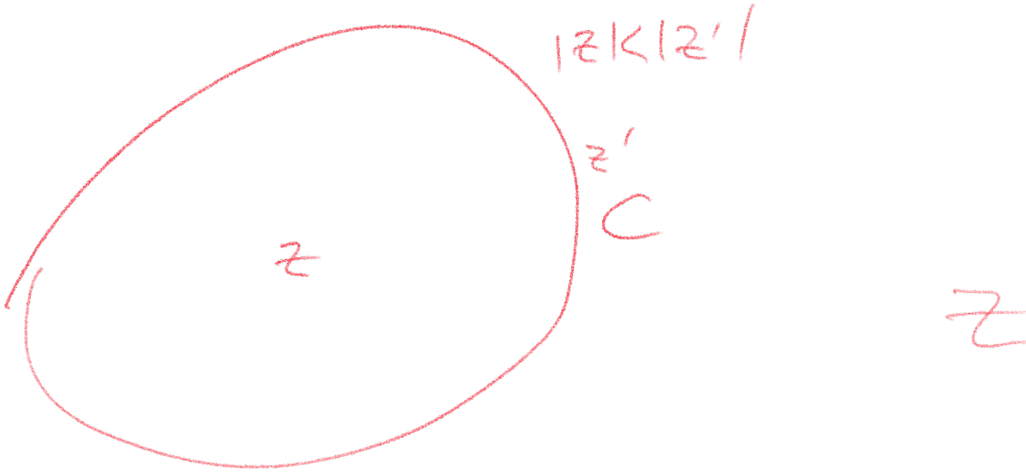


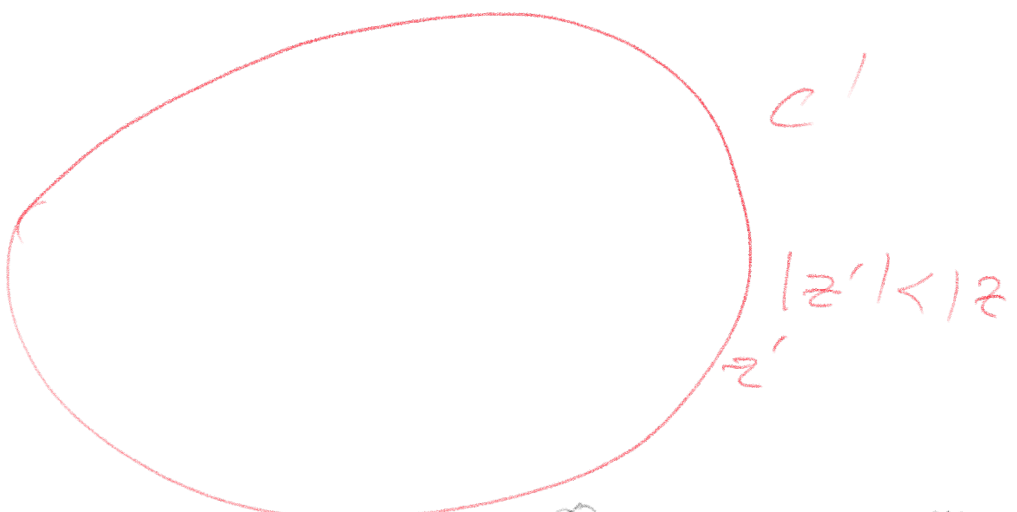
$$= \frac{1}{2\pi i} \oint_C \sum_{n=0}^{\infty} \frac{z^n}{z'^{\,n+1}} e^{\frac{x}{2}\left(z' - \frac{1}{z'}\right)} \, dz'$$

$$+ \frac{1}{2\pi i} \oint_{C'} \sum_{-\infty}^{-1} \frac{z^n}{z'^{\,n+1}} e^{\frac{x}{2}\left(z' - \frac{1}{z'}\right)} \, dz'$$

$$\frac{x}{2}\left(z' - \frac{1}{z'}\right)$$

$$= \frac{1}{2\pi i}\oint_C \frac{dz'}{z'}\, e^{z'} \qquad z' \qquad \overline{\quad 1 - \frac{z}{z'}}$$

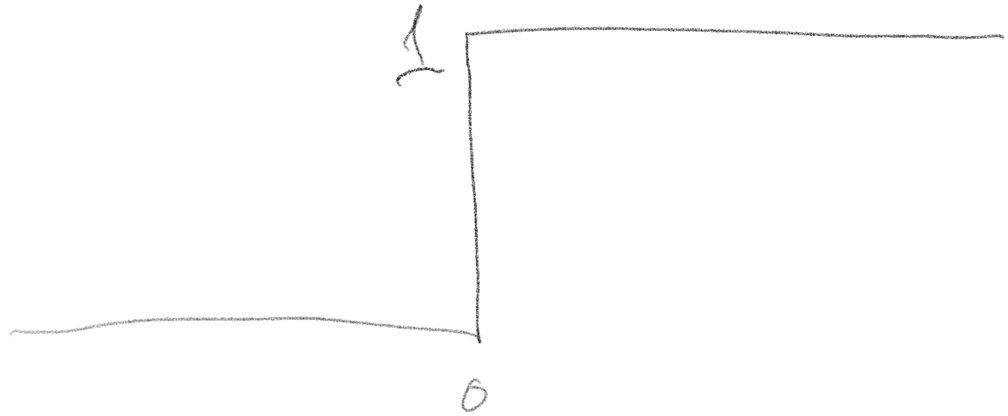




$$\sum_{m=1}^{\infty} \frac{z^{-m}}{z'^{-m+1}} = \sum_{m=1}^{\infty} \frac{1}{z'}\left(\frac{z'}{z}\right)^{m}$$

$$= \sum_{n=0}^{\infty} \frac{1}{z'}\left(\frac{z'}{z}\right)^{n+1} = \frac{1}{z}\sum_{0}\left(\frac{z'}{z}\right)^{n}$$

$$= \frac{1}{z\left(1-\frac{z'}{z}\right)} = z - z'$$



$$\theta(x) = \begin{cases} 0 & x<0 \\ 1 & x>0 \end{cases}$$

$$\delta(x) = \int e^{\pm ikx}\,\frac{dk}{2\pi}$$

$$\left(m^2 + \frac{\partial^2}{\partial t^2} - \nabla^2\right) \Delta_F(x)$$

$$= \int d^4q \; \frac{m^2 - q^{0\,2} + \vec{q}^{\,2}}{\cdots} \; e^{iq\cdot x} = \delta^4(x)$$

$$\int \frac{}{(2\pi)^4} \quad m^2 - q^{0} + \vec{q} - i\epsilon$$

$$(m^2 - \Box)\,\Delta_F(x) = \delta^{(4)}(x)$$

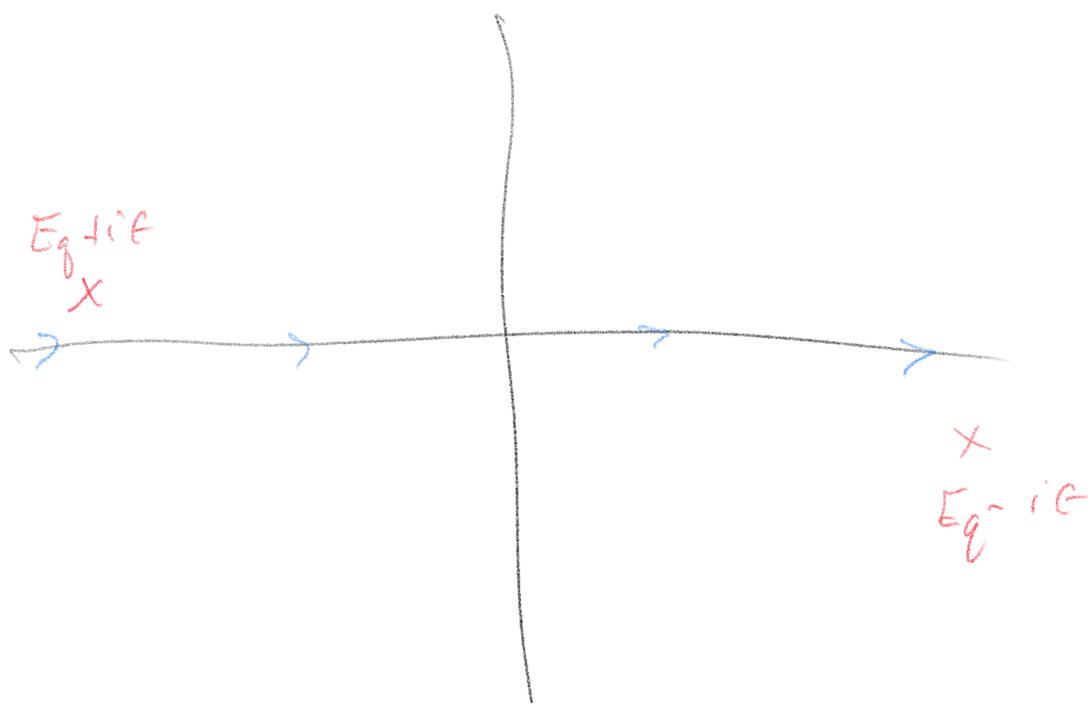
$$\underbrace{\vec{q}^{\,2} - q^{0\,2}}_{-i\epsilon} + m^2 = (E_q - i\epsilon' - q^0)(E_q - i\epsilon' + q^0) + \epsilon'^2$$

$$E_q = \sqrt{m^2 + \vec{q}^{\,2}} > 0$$

$$-i\epsilon'(E_q + q^0 + E_q - q^0) = -2i\epsilon' E_q$$

$$-i\epsilon = -2iE_q\epsilon'$$

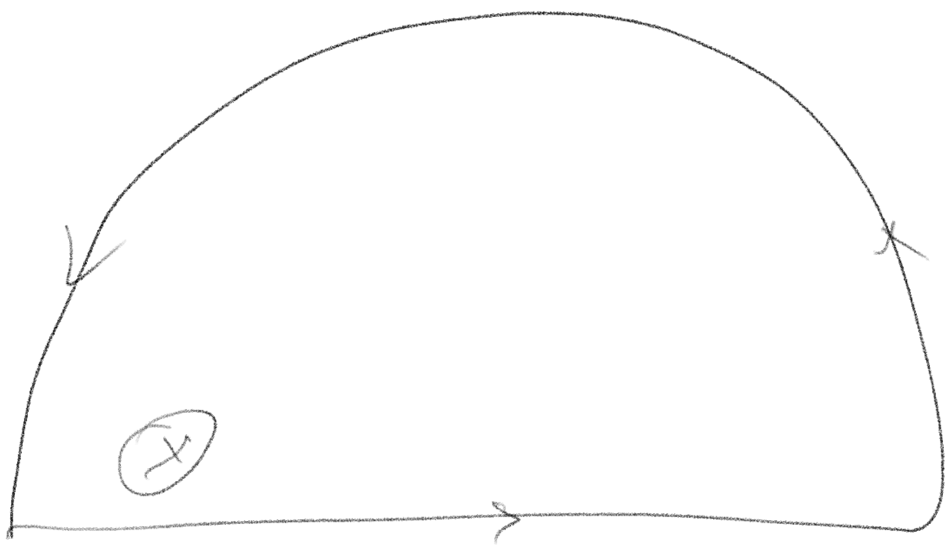
q^0



$x^0 > 0$

$q^0 = iR$

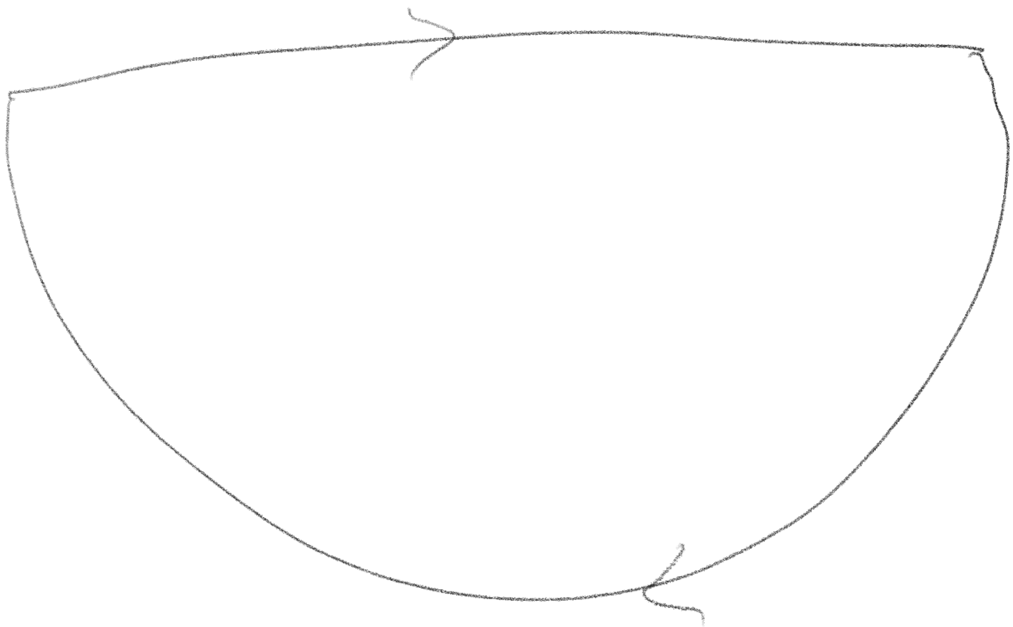
$e^{iq^0x^0} = e^{-Rx^0} \to 0$



$$I(q) = -\int_{-\infty}^{\infty} \frac{dq^0}{2\pi} e^{-iq^0x^0} \frac{1}{[q^0-(E_q-i\epsilon)][q^0-(-E_q+i\epsilon)]}$$

$$= i\,\frac{e^{-iE_qx^0}}{2E_q}$$

$x^0 < 0$



$$I(q) = \frac{i e^{iE_q x^0}}{2E_q} \quad \text{for } x^0 < 0$$