in which x stands for $x_1, \ldots, x_k$ is a linear **partial** differential equation of order $n = n_1 + \cdots + n_k$ in the k variables $x_1, \ldots, x_k$. (A partial differential equation is a whole differential equation that has partial derivatives.)

Linear combinations of solutions of a linear homogeneous partial differential equation also are solutions of the equation. So if f_1 and f_2 are solutions of $L\, f = 0$, and a_1 and a_2 are constants, then $f = a_1 f_1 + a_2 f_2$ is a solution since $L\, f = a_1\, L\, f_1 + a_2\, L\, f_2 = 0$. Additivity of solutions is a property of all linear homogeneous differential equations, whether ordinary or partial.

The **general** solution $f(x) = f(x_1, \ldots, x_k)$ of a linear homogeneous partial differential equation (6.15) is a sum $f(x) = \sum_j a_j f_j(x)$ over a complete set of solutions $f_j(x)$ of the equation with arbitrary coefficients a_j. A linear partial differential equation $L\, f_i(x) = s(x)$ with a source term $s(x) = s(x_1, \ldots, x_k)$ is an **inhomogeneous** linear partial differential equations because of the added source term.

Just as with ordinary differential equations, the difference $f_{i1} - f_{i2}$ of two solutions of the inhomogeneous linear partial differential equation $L\, f_i = s$ is a solution of the associated homogeneous equation $L\, f = 0$ (6.15)

$$L\, [f_{i1}(x) - f_{i2}(x)] = s(x) - s(x) = 0. \tag{6.16}$$

So we can expand this difference in terms of the complete set of solutions f_j of the homogeneous linear partial differential equation $L\, f = 0$

$$f_{i1}(x) - f_{i2}(x) = \sum_j a_j f_j(x). \tag{6.17}$$

Thus the general solution of the inhomogeneous linear partial differential equation $L\, f = s$ is the sum of a particular solution f_{i2} of $L\, f = s$ and the general solution $\sum_j a_j\, f_j$ of the associated homogeneous equation $L\, f = 0$

$$f_{i1}(x) = f_{i2}(x) + \sum_j a_j f_j(x). \tag{6.18}$$

6.3 Notation for Derivatives

One often uses primes or dots to denote derivatives as in

$$f' = \frac{df}{dx} \quad \text{or} \quad f'' = \frac{d^2 f}{dx^2} \quad \text{and} \quad \dot{f} = \frac{df}{dt} \quad \text{or} \quad \ddot{f} = \frac{d^2 f}{dt^2}.$$

For higher or partial derivatives, one sometimes uses superscripts

$$f^{(k)} = \frac{d^k f}{dx^k} \quad \text{and} \quad f^{(k,\ell)} = \frac{\partial^{k+\ell} f}{\partial x^k \partial y^\ell} \tag{6.19}$$