1.34 Show that the totally antisymmetric Levi-Civita symbol ϵ_{ijk} satisfies the useful relation

$$\sum_{i=1}^{3} \epsilon_{ijk}\, \epsilon_{inm} = \delta_{jn}\, \delta_{km} - \delta_{jm}\, \delta_{kn}. \tag{1.449}$$

1.35 Consider the hamiltonian

$$H = \tfrac{1}{2}\hbar\omega\sigma_3 \tag{1.450}$$

where σ_3 is defined in (1.420). The entropy S of this system at temperature T is

$$S = -k\mathrm{Tr}\left[\rho \ln(\rho)\right] \tag{1.451}$$

in which the density operator ρ is

$$\rho = \frac{e^{-H/(kT)}}{\mathrm{Tr}\left[e^{-H/(kT)}\right]}. \tag{1.452}$$

Find expressions for the density operator ρ and its entropy S.

1.36 Find the action of the operator $\boldsymbol{S}^2 = \left(\boldsymbol{S}^{(1)} + \boldsymbol{S}^{(2)}\right)^2$ defined by (1.419) on the four states $|\pm\pm\rangle$ and then find the eigenstates and eigenvalues of $\boldsymbol{S}^2$ in the space spanned by these four states.

1.37 A system that has three fermionic states has three creation operators $a_i^\dagger$ and three annihilation operators a_k which satisfy the anticommutation relations $\{a_i, a_k^\dagger\} = \delta_{ik}$ and $\{a_i, a_k\} = \{a_i^\dagger, a_k^\dagger\} = 0$ for $i, k = 1, 2, 3$. The eight states of the system are $|t, u, v\rangle \equiv (a_1^\dagger)^t (a_2^\dagger)^u (a_3^\dagger)^v |0, 0, 0\rangle$. We can represent them by eight 8-vectors each of which has seven 0's with a 1 in position $4t + 2u + v + 1$. How big should the matrices that represent the creation and annihilation operators be? Write down the three matrices that represent the three creation operators.

1.38 Show that the Schwarz inner product (1.430) is degenerate because it can violate (1.79) for certain density operators and certain pairs of states.

1.39 Show that the Schwarz inner product (1.431) is degenerate because it can violate (1.79) for certain density operators and certain pairs of operators.

1.40 The coherent state $|\{\alpha_k\}\rangle$ is an eigenstate of the annihilation operator a_k with eigenvalue α_k for each mode k of the electromagnetic field,