The First 14 Billion Years

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Abstract

Our universe started nearly 14 billion years ago in a plasma hotter than any nuclear explosion and has been cooling and expanding ever since. After four minutes, the temperature of the plasma had dropped below a billion degrees, and the quarks and gluons of the plasma had turned into protons, neutrons, and helium nuclei. The energy of the plasma, however, remained radiation, mostly photons, neutrinos, and other light particles. After 51,000 years, the energy density of ordinary and dark matter began to outweigh that of radiation. When 380 thousand years had passed, the temperature dropped below 3000 degrees and hydrogen atoms were stable for the first time. The universe suddenly became transparent. About 3.6 billion years ago the dark-energy density exceeded that of matter, and the expansion of the universe accelerated.

Outline of talk

- The big bang
- The first few minutes
- Dark matter
- General relativity
- The Robertson-Walker metric and the scale factor
- Einstein's equations
- The era of radiation
- The era of matter (mostly dark matter)
- Transparency and the cosmic microwave background radiation
- The era of dark energy
- The first 50 billion years
- Before the big bang: inflation

The big bang

The English astronomer Fred Hoyle, who wrote the three best science-fiction books, coined the phrase, "The Big Bang," to make fun of a theory that was a rival to his theory of continuous creation. We don't know what the maximum temperature was during the big bang, but it surely was much hotter than a trillion degrees Kelvin, probably hotter than 10^{16} K or about 1 TeV.
(The conversion factor is

$$1\text{eV} = \frac{e}{k}\,\text{K} = 1.16 \times 10^4\,\text{K} \tag{1}$$

where e is the charge of the electron and k is Boltzmann's constant.) I'll say where this huge energy may have come from later in this talk.

The first few minutes: temperatures and distances

During the first few minutes, the temperature rapidly dropped

$$T \approx \frac{10^{10}\,\mathrm{K}}{\sqrt{t}} \tag{2}$$

where t is the time measured in seconds.

The distance $a(t)$ between objects rapidly expanded as the square-root of the time

$$a(t) \approx a(t')\sqrt{\frac{t}{t'}}. \tag{3}$$

The first few minutes: energies

The energy density for the first 51,000 years was mostly radiation, and so it varied as the fourth power of the temperature T. The equivalent mass density ($m = E/c^2$) of a gas of photons at temperature T is

$$\rho_\gamma = \frac{8\pi^5 (k_B T)^4}{15 h^3 c^5} \tag{4}$$

which is 4.65×10^{-31} kg m^{-3} at the present temperature $T_0 = 2.7255 \pm 0.0006$ K. A few seconds after the big bang, the energy density ρ of the photons, electrons, positrons, and their neutrinos and antineutrinos was

$$\rho \approx 3.8 \times 10^9 \times \left(\frac{T}{10^{11}\text{ K}} \right)^4 \text{ kg/liter} \tag{5}$$

decreasing as the inverse of the square of the time, $1/t^2$.

The first few minutes: nuclei

Free neutrons decay in about a quarter of an hour (881.5 s). After about 3 minutes and 46 seconds, the temperature had dropped enough below 10^9 K that both deuterium (d = (n,p)) and helium (He = (2n,2p)) were stable. The ordinary matter was then about 75 percent protons and 25 percent helium nuclei.
Most of the matter (84.3 %) then was (and now is) of an unknown kind called **dark matter** because it interacts poorly or not at all with light.

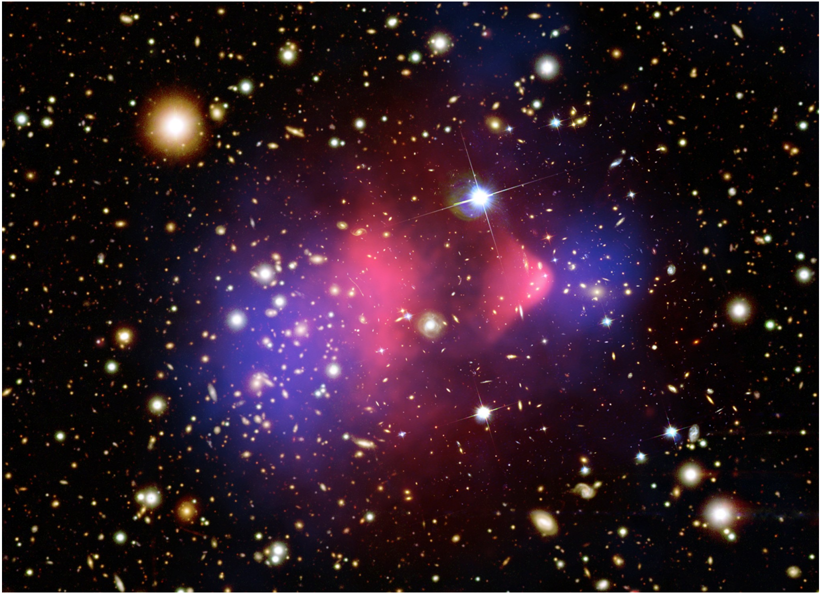
Dark matter

Jacobus Kapteyn in 1922, Jan Oort in 1932, and Fritz Zwicky in 1933 were the first to suggest that much of the matter in the universe was of a kind that we can't see. Zwicky applied the virial theorem which relates the long-term time average $\langle T \rangle$ of the kinetic energy of particles trapped in a $1/r$ potential to the long-term time average of their potential energy $\langle V \rangle$

$$\langle T \rangle = -\frac{1}{2} \langle V \rangle. \tag{6}$$

He estimated that the Coma cluster of galaxies had to have 400 times as much dark matter as visible matter.

The Bullet Cluster



Gravitational lensing

General relativity

The basic ideas of general relativity are:

- The laws of physics are the same in all coordinate systems.
- In every coordinate system, the squared length

$$ds^2 = \sum_{i=0}^{3} \sum_{j=0}^{3} g_{ij}\, dx^i dx^j \qquad (7)$$

is the same. Here $dx^0 = c\,dt$, and g_{ij} is the spacetime metric.

- No local gravity in free-fall coordinates. (Think elevators.)

The Robertson-Walker metric and the scale factor

On the largest scales of distance, the universe is the same in all directions (isotropic) and at all places (homogeneous). In such universes, the metric g_{ij} is simple, and the invariant squared distance is

$$ds^2 = -c^2\,dt^2 + a^2(t)\,\left[\frac{dr^2}{1 - k\,r^2} + r^2\,\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\right]. \quad (8)$$

Here the dimensionless function of time $a(t)$ is the **scale factor**, and k is a constant whose dimension is inverse squared length. If k is negative, the universe is spatially infinite (and open); if k is positive, the universe is finite (and closed). If k is zero, which it may well be, the universe is infinite and flat, and apart from the scale factor $a(t)$ its metric is that of special relativity $ds^2 = a^2(t)\left(dx^2 + dy^2 + dz^2\right) - c^2dt^2$.

Einstein's equations

For the Robertson-Walker metric of a spatially isotropic and homogeneous universe, Einstein's equations relate the square of the Hubble rate $H = \dot{a}/a$ to the energy density ρ

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\,\rho - \frac{c^2 k}{a^2} \tag{9}$$

where $G = 6.674 \times 10^{-11}\ \mathrm{m}^3/(\mathrm{kg\ s}^2)$ is Newton's constant.
I will set $k = 0$ in most of what follows so that

$$\frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\,\rho}\,. \tag{10}$$

The era of radiation

During the first 52 thousand years, the energy density ρ was mostly that of radiation and so it was inversely proportional to the fourth power of the scale factor

$$\rho = \rho_0 \left(\frac{a_0}{a}\right)^4. \tag{11}$$

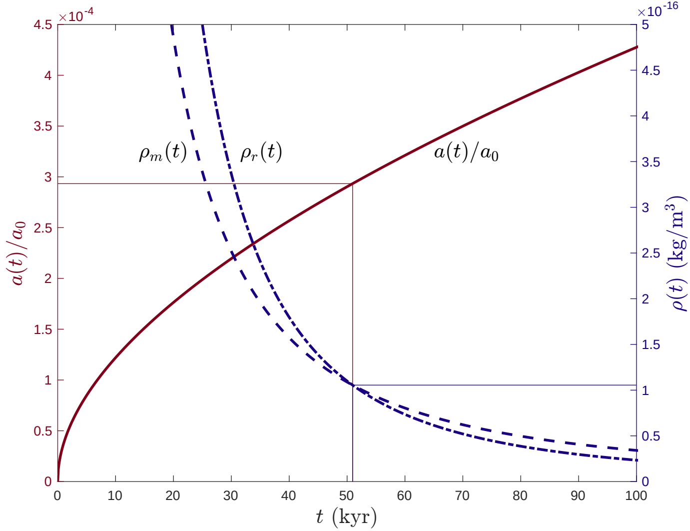
The last two equations give

$$\dot{a} = b/a \quad \text{or} \quad a\,da = b\,dt \tag{12}$$

so

$$a(t) = \sqrt{2b\,t}. \tag{13}$$

Evolution of the scale factor during the era of radiation



The era of matter (mostly dark matter)

If we assume that the particles of matter are stable, then their energy density varies as

$$\rho = \rho_0 \left(\frac{a_0}{a} \right)^3 . \tag{14}$$

Now instead of (12), we have

$$\dot{a} = b' / \sqrt{a} \quad \text{or} \quad \sqrt{a}\, da = b'\, dt \tag{15}$$

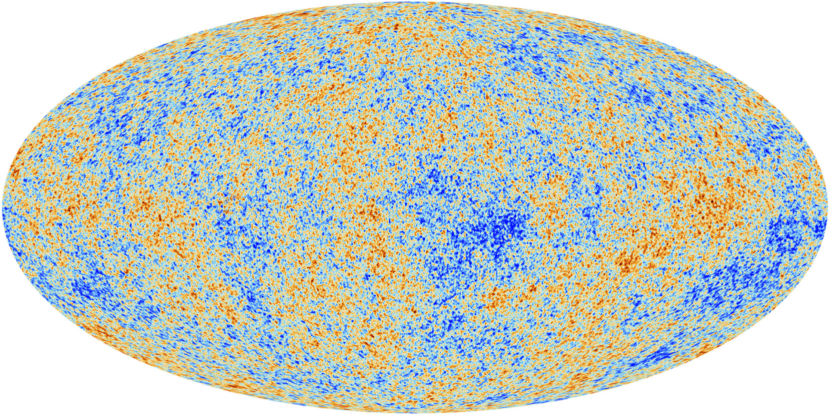
which we integrate to

$$a(t) = \left(\frac{3}{2} b'\, t \right)^{2/3} . \tag{16}$$

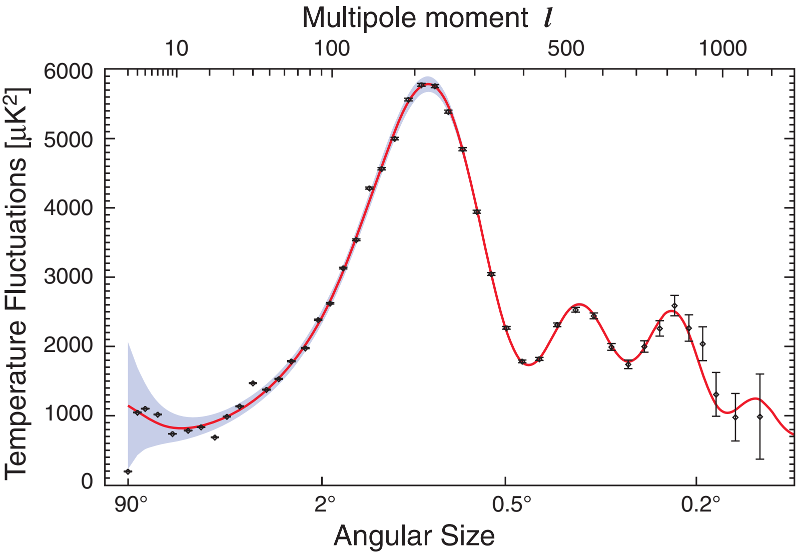
So the scale factor increases somewhat more rapidly during the era of matter.

The era of matter: transparency

At $t = 380,000$ years, the temperature dropped below 3000 K. Hydrogen atoms became stable and the universe became transparent.



The era of matter: the CMBR



The era of dark energy

About 3.6 billion years ago or 10.2 billion years after the big bang, the density of matter dropped below the density of empty space

$$\rho_\Lambda = 5.96 \times 10^{-27} \text{ kg/m}^3 \tag{17}$$

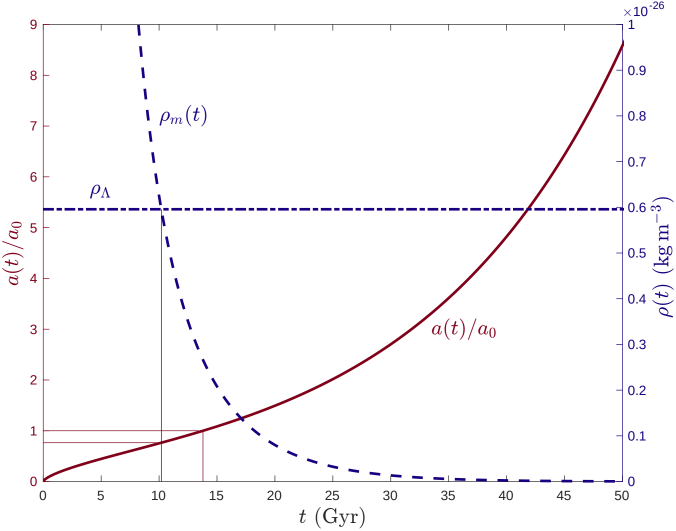
and the era of matter ended. Because the vacuum energy density ρ_Λ is constant, Einstein's equations are particularly simple during the era of dark energy

$$\dot{a} = H_0\, a \quad \text{or} \quad a(t) = a(0)e^{H_0\, t} \tag{18}$$

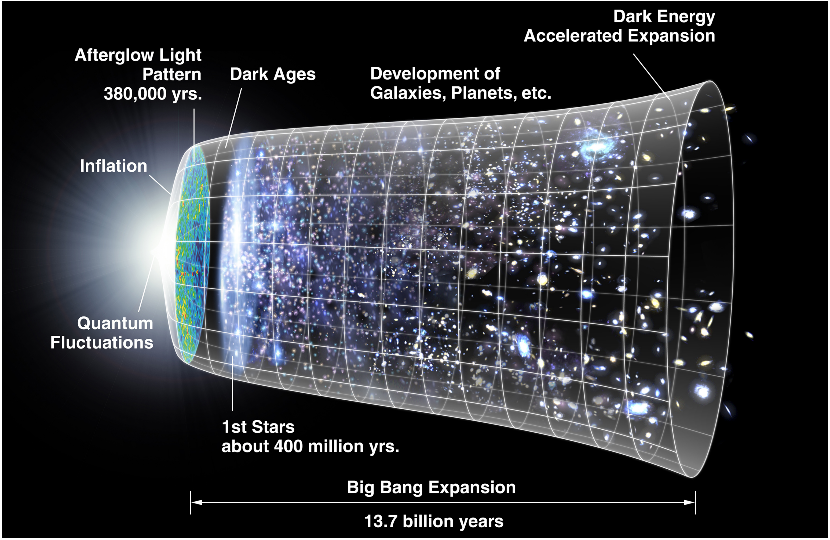
in which H_0 is the present value of the Hubble rate

$$H_0 = 67.74 \text{ km/(s Mpc)} = 2.1953 \times 10^{-18}\text{s}^{-1}. \tag{19}$$

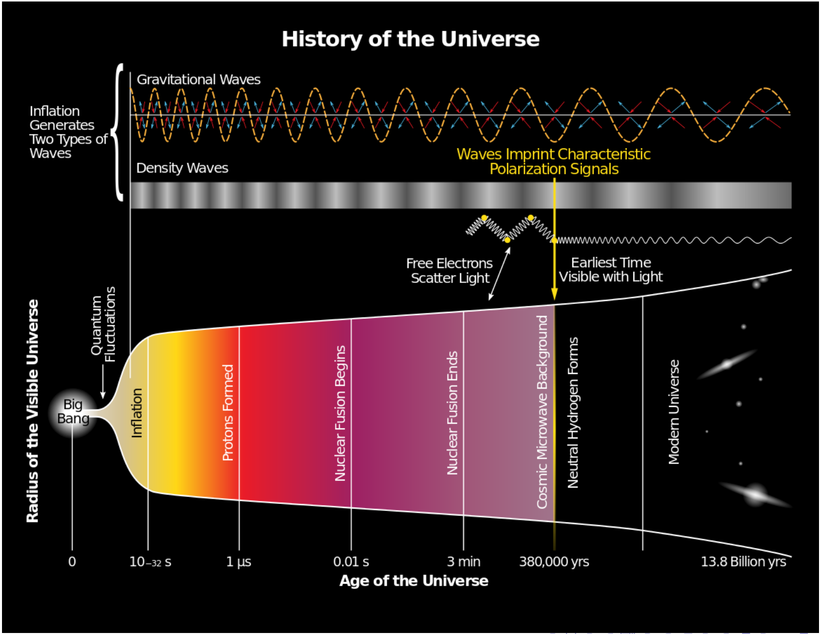
The first 50 billion years



The first 13.7 billion years



The first 13.8 billion years by Yinweichen



Inflation

Scalar fields fluctuate. Once upon a time, a scalar field $\phi(x)$ underwent a quantum fluctuation to a value at which its potential-energy density $m^2\phi^2(x)$ was huge. General relativity then imposed what we may call gravitational friction, and so the return of the field to its low-energy value $\phi(x) = 0$ was not instantaneous. During that brief interval, the energy density of the vacuum was $\rho = m^2\phi^2(x)$, and so the scale factor $a(t)$ expanded as

$$a(t) = a(0) \exp\left(\sqrt{(8\pi G m^2/3)\,\phi^2(x)}\;\, t\right). \tag{20}$$

Alan Guth called this **inflation**; Andrei Linde called it chaotic inflation or eternal inflation. Inflation explains why the universe is nearly flat and why the CMBR is nearly the same temperature in all directions. Energy was conserved during and after inflation.

Physical Mathematics

Unique in its clarity, examples, and range, *Physical Mathematics* explains as simply as possible the mathematics that graduate students and professional physicists need in their courses and research. The author illustrates the mathematics with numerous physical examples drawn from contemporary research. In addition to basic subjects such as linear algebra, Fourier analysis, complex variables, differential equations, and Bessel functions, this textbook covers topics such as the singular-value decomposition, Lie algebras, the tensors and forms of general relativity, the central limit theorem and Kolmogorov test of statistics, the Monte Carlo methods of experimental and theoretical physics, the renormalization group of condensed-matter physics, and the functional derivatives and Feynman path integrals of quantum field theory.

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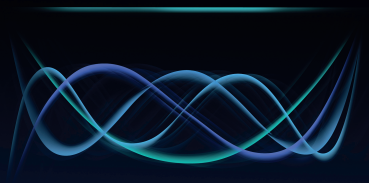

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