The Main Ideas of Science

Kevin Cahill
Department of Physics and Astronomy
University of New Mexico, Albuquerque, NM 87131-1156

April 12, 2010

For Marie, Mike, Sean, and Peter,
and in honor of Mr. Muntader al-Zaidi.

Contents

Preface

A word to students: These class notes are incomplete; in fact, I am only now starting to write them.

Chapter 1

Once Upon a Time

1.1 The Big Bang

About 13.73 billion years ago (that's 13,730,000,000 BC), the observable universe started in a **Big Bang**. What happened in the first instants of that explosion is uncertain, and what if anything happened before the Big Bang is unknown.

It is likely that in less than a nanosecond = 10^{-9} s = 0.000000001 s, the visible universe rapidly expanded from smaller than an atom to bigger than a basketball. This expansion is called **inflation**. More about it later.

At the end of the instant of inflation, the universe became extremely hot and exploded in what is called the **Big Bang**. The universe has been expanding since then, that is for the last 13.73 billion years. This post-Big-Bang expansion at first was very fast (but much slower than during the instant of inflation); it then slowed down due to the gravity until about 4 billion years ago, when it began to speed up due to a mysterious energy density called **dark energy**, which makes up about 72.1% of the energy density of the universe. The kind of matter we know about makes up only about 4.6% of the energy density of the universe, while an unknown kind of matter that does not interact with light and so is called **dark matter** makes up another 23.3%. More about dark energy and dark matter later.

Steven Weinberg picks up the story at 1/100th of a second after the Big Bang in his marvelous book *The First Three Minutes* [1]. Before summarizing that book, which you are expected to read, let me tell you a bit about the smallest and most basic of the particles we know of, especially those that play roles in his book.

1.2 The Particles We Know

These **elementary particles** are first of all two **leptons**, the **electron** e and its **neutrino** ν_e, three **up quarks** u_r, u_g, u_b, and three **down quarks** d_r, d_g, d_b. The subscripts r, g, and b stand for *red*, *green*, and *blue*, but these are only fanciful labels for the three kinds of up and down quark, not actual colors. The electron is negatively charged. The size of its charge is the basic unit of electric charge $e = 1.6 \times 10^{-19}$ C, where C stands for a much larger unit of charge called the Coulomb (named after Charles-Augustin de Coulomb, 1736–1806). Each of the three up quarks carries charge $2e/3$, and each of the down quarks carries charge $-e/3$. The neutrino is electrically neutral. In the **Standard Model of Particle Physics**, these leptons and quarks often are displayed as

$$\begin{pmatrix} \nu_e & u & u & u \\ e & d & d & d \end{pmatrix}. \tag{1.1}$$

Next come the **anti-particles**: the electron, its neutrino, and each of the six quarks has a partner that has the same mass but the opposite charge. These eight particles and their anti-particles

$$\begin{pmatrix} \bar{e} & \bar{d} & \bar{d} & \bar{d} \\ \bar{\nu}_e & \bar{u} & \bar{u} & \bar{u} \end{pmatrix} \tag{1.2}$$

form the first of three **families** (or **generations**). The anti-particle of the electron is called the **positron**. The masses of the neutrinos and of the up and down quarks are not known exactly.

The second and third families of leptons and quarks are unstable; they do not occur in ordinary matter unless it is extremely hot. In the second family of quarks and leptons, the **muon** (symbol μ) replaces the electron, and its neutrino, the ν_μ, replaces the electron neutrino; three **charmed** quarks replace the three up quarks; and three **strange** quarks replace the three down quarks:

$$\begin{pmatrix} \nu_\mu & c & c & c \\ \mu & s & s & s \end{pmatrix}. \tag{1.3}$$

The second family of anti-quarks and anti-leptons is

$$\begin{pmatrix} \bar{\mu} & \bar{s} & \bar{s} & \bar{s} \\ \bar{\nu}_\mu & \bar{c} & \bar{c} & \bar{c} \end{pmatrix}. \tag{1.4}$$

In the third family of quarks and leptons, the **tau** lepton replaces the electron, and its neutrino ν_τ replaces the electron neutrino; three **top** quarks and three **bottom** quarks replace the up and down quarks:

$$\begin{pmatrix} \nu_\tau & t & t & t \\ \tau & b & b & b \end{pmatrix}. \tag{1.5}$$

Their anti-particles are:

$$\begin{pmatrix} \bar{\tau} & \bar{b} & \bar{b} & \bar{b} \\ \bar{\nu}_\tau & \bar{t} & \bar{t} & \bar{t} \end{pmatrix}. \tag{1.6}$$

The particles of the second and third families are exactly like those of the first family—**as far as we know**—apart from differences in mass. The particles of the third family are heavier than those of the second family, which are heavier than those of the first family.

All 48 of these elementary particles carry the smallest possible non-zero amount of angular momentum; they are said to have **spin one-half** and are called **fermions** (after Enrico Fermi, 1901–1954).

If these 48 quarks, leptons, and their anti-particles were all there were, they would be massless and would move freely in straight lines at the speed of light

$$c = 300,000 \quad \text{km/s} \tag{1.7}$$

or 186,000 miles per second. In particular, they would pass by or through each other without slowing down or interacting.

But these 48 particles do have mass and do interact with each other and with other elementary particles. In the standard model, their masses arise from their interactions with other particles. They interact by the exchange of particles of spin 0, 1, and 2 called **bosons** (after Satyendra Bose, 1894 – 1974). One of these bosons is the **photon**; it is massless and has spin 1. It responds to electric charge and mediates electric and magnetic interactions. Without the photon, there would be no TV, no computers, no electricity or magnetism, no radio, no internet; worse yet, there would be no atoms or molecules, and so no life at all. Some of the basic physical processes involving photons, electrons, and positrons are sketched as Feynman diagrams in Fig. 1.1.

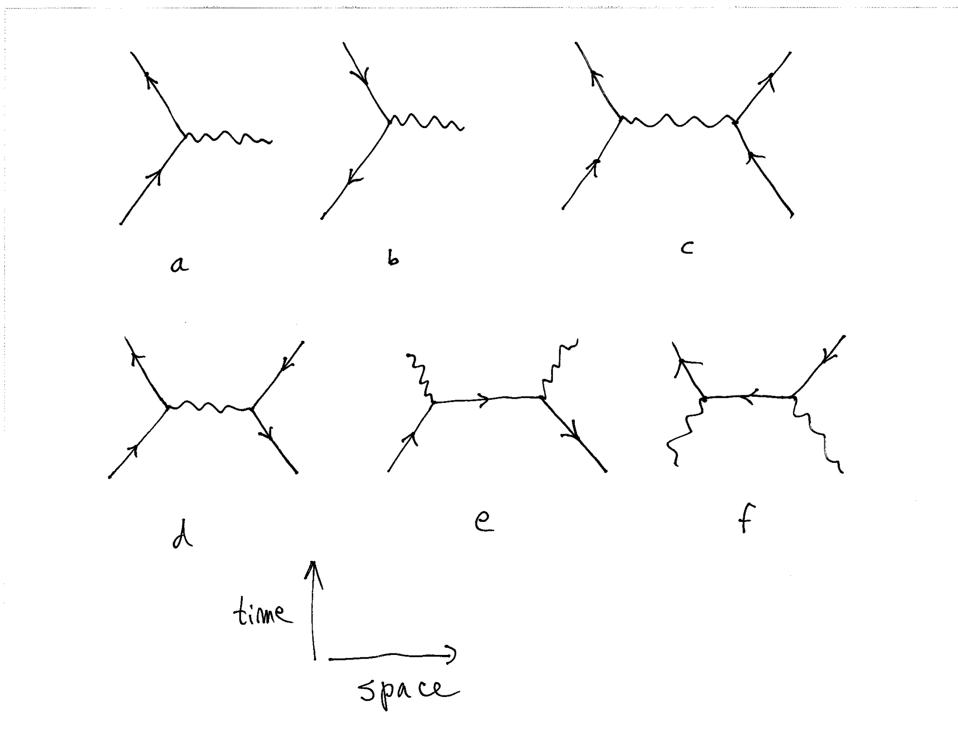


Figure 1.1: Feynman diagrams of some elementary electromagnetic interactions: a, an electron emits or absorbs a photon; b, a positron emits or absorbs a photon; c, two electrons exchange a photon; d, an electron and a positron exchange a photon; e, an electron and a positron turn into two photons; and f, two photons become an electron and a positron. In these diagrams, time runs up, and space runs horizontally.

Another is the **graviton**; is is massless, has spin 2, responds to mass and energy ($E = mc^2$), and mediates gravity. These are the only known massless particles. In empty space, they move at the speed of light, but they do interact. The photon interacts with charged particles; the graviton interacts with all particles. Because they are massless, the photon and graviton carry electromagnetic and gravitational interactions over long distances. For instance, gravitons keep the Earth orbiting the Sun—without them it would sail off into outer space in a straight line. The photons we see in the night sky come from stars that are at least 4 light-years away. One light-year is the distance light travels in one year going at 300,000 km/s. There are about 31 million seconds in a year, so one light-year is 9 trillion km

$$\begin{aligned} ly &= c \times 31,560,000\,\mathrm{s} = 300,000\,\frac{\mathrm{km}}{\mathrm{s}} \times 31,560,000\,\mathrm{s} \\ &= 9,000,000,000,000\,\mathrm{km} = 9.47 \times 10^{12}\,\mathrm{km}. \end{aligned} \tag{1.8}$$

Three other spin-one bosons—the positive W^+ of charge e, its antiparticle, the W^- of charge $-e$, and the neutral Z—mediate the **weak interactions**. These interactions actually are as strong as the electromagnetic interaction, but they are of very short range because the $W^\pm$ and the Z are about as massive as an atom of the noble gas Krypton.

Eight neutral, colorful, massless, spin-one **gluons** mediate the **strong** force. This force **confines** quarks and gluons in **colorless** blobs called **hadrons**. The **proton**, for example, consists of two up quarks, a down quark, and many gluons; its charge is therefore

$$\frac{2}{3}\,e + \frac{2}{3}\,e - \frac{1}{3}\,e = e \tag{1.9}$$

equal to e, that is, the charge of the proton is exactly the opposite of the charge of the electron. The other main constituent of atomic nuclei is the **neutron**; it is made of one up quark and two down quarks, so its charge is zero

$$\frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0. \tag{1.10}$$

No matter how hard two protons are slammed into each other, the quarks and gluons are never shaken out as freely moving particles. This **confinement** of quarks and gluons is one of the most robust and striking phenomena of all of physics; it is so poorly understood as to be an embarrassment. Incidentally,

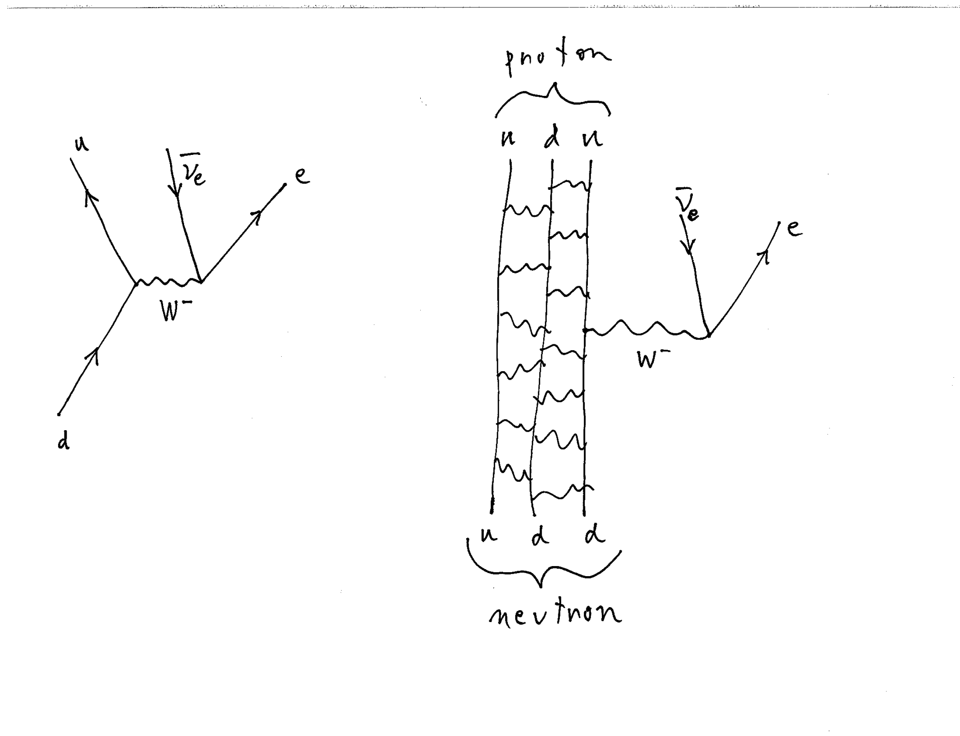


Figure 1.2: Feynman diagrams of (left) a down quark emitting a W^- and changing into an up quark while the emitted W^- decays into an electron and an anti-neutrino, and (right) the decay of a neutron into a proton, an electron, and an anti-neutrino. The many squiggles connecting the *udd* quarks of the neutron and the *udu* quarks of the proton represent the gluons that hold these quarks together and that these quarks exchange.

the confinement of quarks inside **hadrons** is why the masses of the lighter quarks are uncertain.

In the **standard model** of high-energy physics, there is one more particle, a neutral spin-zero boson called the **Higgs boson**. In the standard model, all particles get their masses due to their interactions with this particle, the Higgs boson. It has not yet been seen, so we don't know what its mass is.

1.3 Temperature

Some words about temperature. The physical scale of temperature is the Kelvin scale in which $T = 0$ K is absolute zero, the temperature at which atoms and molecules stop moving about. The centigrade scale has degrees of the same size, but $T = 0$ C (at which ice melts) is the same as $T = 273.15$ K. Fahrenheit degrees are smaller by the factor 5/9, and $T = 32$ F is the same as $T = 273.25$ K.

An object at temperature T radiates and absorbs photons from its environment. The average energy of the radiated photons is about $E = 2.7kT$ in which $k = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K, in which eV means electron-volt, which is the energy an electron gains when it drops through a potential difference of one Volt.

1.4 The Expansion of the Universe

During those three minutes, the universe evolved from a very hot expanding gas of all kinds of elementary particles—electrons, positrons, photons, neutrinos, quarks, gluons, *etc.*—into a gas of photons, neutrinos, electrons, protons, and the nuclei of helium atoms plus traces of other nuclei.

How much has the universe expanded? The universe is believed to have expanded by at least a factor of $F_i = 10^{26}$ during the very brief phase of inflation. At the end of that phase, the vacuum energy that drove inflation was converted into radiation, and the temperature of the universe rose in the Big Bang to an unknown but exceedingly high temperature. In most models of inflation, the Big Bang temperature T_{BB} is somewhere between 10^{16} and 10^{23} K (degrees Kelvin). The universe then expanded during an era of radiation for about 65,000 years until the temperature had dropped to about $T_r = 12,000$ K. During this era the universe expanded by a factor of

$F_r = T_{BB}/T_r$ plausibly between $10^{16}/10^4 = 10^{12}$ and $10^{23}/10^4 = 10^{19}$.

As the universe expanded, the wavelengths of these photons also expanded. The **red-shift** z relates the stretched wave-length λ_0 now seen to the wave-length λ of the emitted light and also to the expansion factor F_z of the universe between the emission of the light and the present time

$$1 + z = \frac{\lambda_0}{\lambda} = F_z. \tag{1.11}$$

The red-shift z from 65,000 years to now is about 3000, and so this expansion factor is about $F_z = 3000 = 3 \times 10^3$.

Putting these three expansion factors together, we find that the total expansion F of the universe including inflation was somewhere between

$$F = F_i F_r F_z = 10^{26} \times 10^{12} \times 3 \times 10^3 = 3 \times 10^{41} \tag{1.12}$$

for a 10^{16} K Big Bang

$$F = F_i F_r' F_z = 10^{26} \times 10^{19} \times 3 \times 10^3 = 3 \times 10^{48} \tag{1.13}$$

for a 10^{23} K Big Bang.

The radius R of the visible universe is 13.7 billion years times the speed of light or 13.7 billion light-years or

$$R = 13.7 \times 10^9 \,\text{ly} = 13.7 \times 10^9 \times 9.47 \times 10^{12}\,\text{km} = 1.3 \times 10^{23}\,\text{km}. \tag{1.14}$$

Thus just before inflation, the visible universe would have had a radius R_0 of

$$R_0 = \frac{R}{F} = \frac{1.3 \times 10^{23}\,\text{km}}{3 \times 10^{41}} \approx 4 \times 10^{-19}\,\text{km} = 4 \times 10^{-16}\,\text{m} \tag{1.15}$$

for a 10^{16} K Big Bang and

$$R_0' = \frac{R}{F} = \frac{1.3 \times 10^{23}\,\text{km}}{3 \times 10^{48}} \approx 4 \times 10^{-26}\,\text{km} = 4 \times 10^{-23}\,\text{m} \tag{1.16}$$

for a 10^{23} K Big Bang.

From 65,000 years until about 380,000 years, the universe was an ionized plasma. By 380,000 years, its temperature had dropped to about 3000 K which is cool enough for atoms to be stable. At this temperature, the universe underwent a phase change from an opaque ionized plasma of electrons and nuclei to a transparent gas of hydrogen and helium. In the subsequent 13.7

billion years, the mean temperature of the photons dropped from 3000 K to 2.725 K. As their temperature dropped, the mean wave-length λ of these photons was stretched out by the expansion of the universe from $\lambda_{3000} = 0.0029/3000$ m $= 966$ nm to $\lambda_{2.7} = 0.0029/2.7 = 1.06$ mm or by the ratio

$$\frac{\lambda_{2.7}}{\lambda_{3000}} = \frac{3000}{2.725} \approx 1100. \tag{1.17}$$

The expansion of the universe during this era was therefore by a factor of 1100, but that's included in the factor $F_z = 3000$.

1.5 Stars and Galaxies

The first stars formed some 600 million years after the Big Bang at about $z = 12$. Many were 100 to 500 times as massive as the Sun. Such massive stars have very hot cores and end as supernovas only about a million years after forming. Some of their cores are so hot— $T > 10^{10}$ K—that two core photons can turn into an electron and a positron as in the Feynman diagram of process f of Fig. 1.1. Each such event converts a million eV of energy from radiation to the rest energy of the electron and positron. Thus the radiation pressure in the core drops, and the star collapses. Zillions of nuclei falling into the low-pressure core fuse into heavier nuclei releasing huge amounts of energy in a bright supernova explosion.

At a billion years after the big bang at a z of about 8.5, the first galaxies formed around super-massive black holes.

Chapter 2

Quantum Mechanics

2.1 Physics before 1905

During the 19th century, many scientists were unsure whether atoms and molecules were real particles or just accounting tricks used by chemists. This changed in 1905 when Albert Einstein analyzed the (Brownian) motion of microscopic objects in fluids and related Boltzmann's constant k to a combination of quantities—the absolute temperature T, the viscous-friction coefficient ζ, and the diffusion constant D—that had been or could be measured

$$k = \zeta D/T. \tag{2.1}$$

The present value of Boltzmann's constant k is 1.38×10^{-23} J/K. By this time, people knew that the pressure p and temperature T in degrees Kelvin (K) of n moles (mol's) of an ideal gas in a volume V were related to the known gas constant $R = 8.31$ J/(K mol) by

$$pV = nRT. \tag{2.2}$$

They also knew that $pV = NkT$ in which N is the number of (hypothetical) molecules in the gas. Thus once they knew Boltzmann's constant k, they could determine the number of molecules in a mole as

$$N_A = R/k = 8.31/(1.38 \times 10^{-23}) = 6.02 \times 10^{23} \tag{2.3}$$

known as Avagadro's number. Immediately, they knew that the mass of a molecule whose molecular weight was M grams was just M/N_A grams or

$10^{-3} \times M/N_A$ kg. In particular, an atom of hydrogen was found to have the astonishingly small mass of 1.67×10^{-27} kg.

Having established the reality and the masses of atoms and molecules, Einstein also in 1905 pointed out that light was made of particles (later called photons) whose energy E and momentum p were related to their frequency ν and wavelength λ by

$$E = h\nu \quad \text{and} \quad p = \frac{h}{\lambda} \tag{2.4}$$

in which $h = 6.6 \times 10^{-34}$ Joule seconds (Js).

It is not surprising then that the physical theories developed before 1900 describe macroscopic processes, i.e., ones that we can see and measure directly without much instrumentation. For example, using the theories of optics and mechanics as well as clocks and telescopes, we can measure the position and momentum of an asteroid and then predict whether it will hit the Earth within a few years.

2.2 The Uncertainty Principle

It is not so easy, however, to measure the position and momentum of a microscopic particle. The photons from the Sun that bounce off an asteroid and enter a telescope have a negligible effect upon the orbit of the asteroid. But what if the momentum of the microscopic particle is no bigger then that of the photons we use to observe it? In this case, the photons we use to measure its position will significantly alter its momentum. Werner Heisenberg realized in 1927 that it is impossible to measure simultaneously both the position and the momentum of any particle to arbitrary accuracy. According to his **uncertainty principle** there is a lower limit to the product of the errors Δx and Δp in the simultaneous measurements of the position x and momentum p of any particle. This product of errors (or uncertainties) must exceed Planck's constant, $h = 6.6 \times 10^{-34}$ Joule seconds (Js), divided by 4π

$$\Delta x \, \Delta p \geq \frac{h}{4\pi} = 5.3 \times 10^{-35} \quad \text{Js.} \tag{2.5}$$

The reason this limit was not discovered until 1927 is that it is many orders of magnitude smaller than the errors we expect to make when measuring macroscopic things. Suppose, for example, that we measured to an accuracy of 0.001 meter (m) the position of a mass of 0.001 kilogram (kg) and simultaneously measured its speed to an accuracy of 0.001 m/s. These are fairly precise measurements. Our uncertainty in position would be $\Delta x = 0.001$ m, and our uncertainty in momentum would be $\Delta p = 0.001 \times 0.001 = 10^{-6}$ kg m/s. The product of these quite small errors is $\Delta x \, \Delta p = 10^{-9}$ Js. Yet this product is bigger than Heisenberg's limit $h/(4\pi)$ by a factor of 1.9×10^{25}. But what if we measured the position and speed of an electron to the same precision? The error in position would still be $\Delta x = 0.001$ m, but because the mass of an electron is $m_e = 9.1 \times 10^{-31}$ kg, our error in measuring its momentum would be only $\Delta p = 9.1 \times 10^{-31} \times 0.001 = 9.1 \times 10^{-34}$ kg m/s. The product of our uncertainties would then be $\Delta x \, \Delta p = 9.1 \times 10^{-37}$ kg m/s. But this is less than Heisenberg's limit of 5.3×10^{-35} Js by a factor of 0.017. In other words, it is impossible to measure the position and momentum of an electron as precisely as we were supposing.

We saw in class a demonstration of the uncertainty principle. If a beam of photons from a laser hits a vertical slit of width Δx, then their horizontal position is known to within Δx as they pass through the slit. Heisenberg's uncertainty principle (2.5) then says that the uncertainty in their horizontal momentum will exceed

$$\Delta p \geq \frac{h}{4\pi \Delta x}. \tag{2.6}$$

The narrower the slit, the wider the momentum spread, as illustrated in Fig. 2.1. If one narrows the slit, then the bright spot of photons passing through the slit and striking a screen will widen. If one widens the slit, the spot will narrow and eventually become as narrow as the width of the beam leaving the laser. When the slit was about 0.1 mm = 0.0001 m wide, the bright spot was about 2 cm = 0.02 m wide at a distance of 2 m from the slit. Here $\Delta x = 0.0001$, and Δp is the fractional change in p, that is, $\Delta p = 0.02 \times p/2$. The momentum p is Planck's constant h divided by the wavelength $\lambda = 633$ nm $= 6.33 \times 10^{-7}$ m. So the product of the uncertainties is

$$\Delta x \, \Delta p = 0.0001 \times \frac{0.02h}{2 \times 6.33 \times 10^{-7}} = 1.6h > \frac{h}{4\pi} \tag{2.7}$$

which exceeds Heisenberg's limit as it must.

In 1963, J. Robert Oppenheimer suggested another way of thinking about the uncertainty principle: a device that measures the position of a particle must be fixed in space, while one that measures its momentum must be free to recoil when struck by the particle.

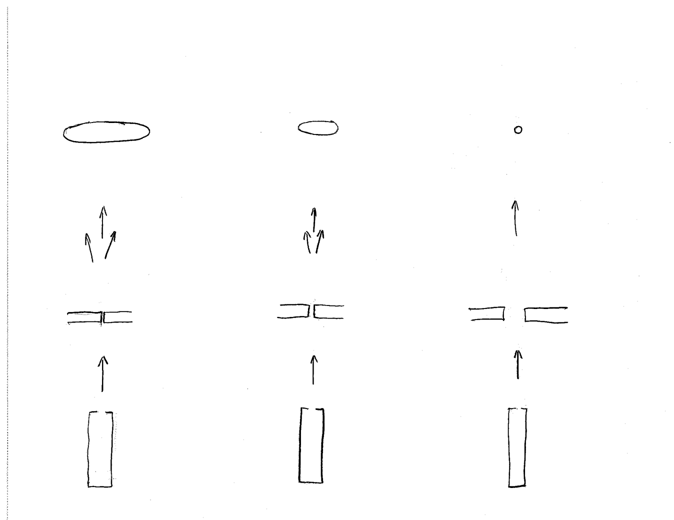


Figure 2.1: Photons from three lasers pass through three slits. Those that pass through the very narrow left slit make a broad bright spot on the screen. Those that pass through the less narrow slit make a less broad spot on the screen. Those that pass through the wide slit make a spot on the screen no bigger than the diameter of the opening of the laser.

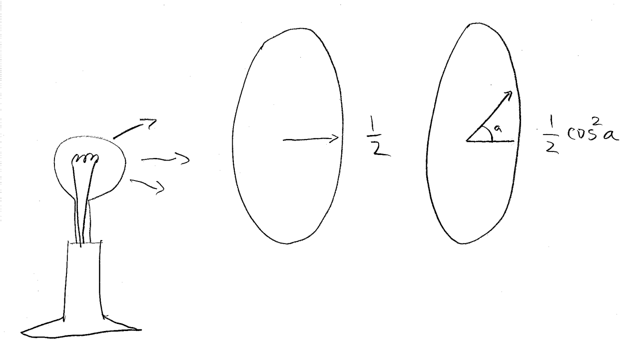


Figure 2.2: Photons from the light bulb are unpolarized. Half of them get through the first (horizontal) polarizing filter. The second polarizing filter makes an angle a with the horizontal. The fraction of photons that pass through both filters is $(1/2)\cos^2 a$.

2.3 Amplitudes and Probabilities

Quantum mechanics is the theory of microscopic phenomena. Since we can't measure the initial position and momentum of a particle simultaneously and exactly, we can't predict its future motion precisely. Thus quantum mechanics is statistical, not deterministic. It provides rules for computing the probability P that an event e' will follow an event e after a time t.

Polarized light provides a simple example. A photon has two possible states of polarization, horizontal and vertical, which are related to its two possible states of spin. Suppose we put an ideal horizontal polarizing filter in front of a source of ordinary unpolarized light. The probability that a particular photon will pass through the filter is $P = 1/2$; on average, half the photons will pass through the filter. Those that do pass through it will be in the horizontal state of polarization, an event e_h.

In general, quantum mechanics associates an amplitude A with the occurrence of an event e' at time t after an event e, and this amplitude is a pair of numbers

$$A(e', e, t) = (x, y). \tag{2.8}$$

The probability $P(e', e, t)$ is then the sum of the squares of these two numbers

$$P(e', e, t) = x^2 + y^2. \tag{2.9}$$

Again polarized light provides an example. Suppose we put a second ideal polarizing filter behind the first one but rotated by an angle a as in Fig. 2.2. Passage through the second filter is event e'_a. What is the probability $P(e'_a, e_h, t)$ that a photon that gets through the first, horizontal filter, event e_h, gets through the filter at angle a, event e'_a, after a suitable delay t? In this case, the amplitude A that e'_a follows e_h is

$$A(e'_a, e_h, t) = (\cos a, 0) \tag{2.10}$$

and so the probability $P(e'_a, e_h, t)$ is

$$P(e'_a, e_h, t) = \cos^2 a. \tag{2.11}$$

Often there are several ways in which an event e' can follow an event e after a time t. In such cases, quantum mechanics tells us to compute the amplitudes $A_1 = (x_1, y_1)$, $A_2 = (x_2, y_2)$, ... for all the processes that lead from e to e' in a time t and add them up to find the total amplitude $A = A(e', e, t)$

$$\begin{aligned} A &= A_1 + A_2 + A_3 + \cdots = (x_1, y_1) + (x_2, y_2) + (x_3, y_3) + \ldots \\ &= (x_1 + x_2 + x_3 + \ldots, y_1 + y_2 + y_3 + \ldots). \end{aligned} \tag{2.12}$$

Quantum mechanics provides many ways of calculating such amplitudes. Richard Feynman's way is direct and uses a physical quantity called the action.

2.4 Action

Every physical process can be assigned a number called the **action** S with the dimensions of energy×time or Joule seconds (Js) in SI units. For instance, a mass of one kilogram moving at one meter per second for two seconds is a process with one Js of action. The basic unit of action is the very much smaller constant introduced by Max Planck, $h = 6.6 \times 10^{-34}$ Js. A process whose action is less than h requires a quantum-mechanical description.

The action for a process in which a particle moves along a path of length ℓ in time t is the average momentum $\langle p \rangle$ along the path times its length minus the time average of the energy $\langle E \rangle$ multiplied by the time

$$S = \langle p \rangle \ell - \langle E \rangle t. \tag{2.13}$$

Here the angular brackets mean that we are to take the average of p and E.

In 1905, Einstein explained the photo-electric effect by assuming that the energy E of a photon is related to its frequency ν by $E = h\nu$ and that its momentum p is related to its wave-length λ by $p = h/\lambda$. Thus the action S of a photon of frequency ν and wave-length λ going a distance ℓ in a time t is

$$S = \langle p \rangle \ell - \langle E \rangle t = \langle h/\lambda \rangle \, \ell - \langle h\nu \rangle t = h \left(\langle 1/\lambda \rangle \, \ell - \langle \nu \rangle t \right). \tag{2.14}$$

If we suppress the explicit representation of the averaging, then this formula takes the simple form

$$S = h \left(\ell/\lambda - \nu t \right) \tag{2.15}$$

which de Broglie extended to massive particles in 1924.

2.5 Feynman's Amplitude Formula

Feynman's formula for the total amplitude of a process $e \to e'$ in time t has three steps:

1. Compute the action S_n of each path ($n = 1, 2, 3, \ldots$), that leads from $e \to e'$ in time t.

2. Then compute an angle $\theta_n = 2\pi S_n/h$ radians (or equivalently $\theta_n = 360 S_n/h$ degrees).

3. The coordinates (x, y) of the point that is at angle θ_n on the unit circle is the un-normalized or relative amplitude a_n for the nth path

$$a_n = (x_n, y_n). \tag{2.16}$$

4. Add all the un-normalized amplitudes

$$a(e', e, t) = (x_1, y_1) + (x_2, y_2) + (x_3, y_3) + \cdots = (x', y'). \tag{2.17}$$

as in (2.12).

5. The relative probability $p(e', e, t)$ that event e' follows event e in time t is
$$p(e', e, t) = x'^2 + y'^2. \tag{2.18}$$

6. Repeat the above process to find the relative amplitudes $a(e'', e, t) = (x'', y'')$, $a(e''', e, t) = (x''', y''')$, etc., and relative probabilities $p(e'', e, t) = x''^2 + y''^2$, $p(e''', e, t) = x'''^2 + y'''^2$, etc., for the other events e'', e''', etc., that can follow event e in time t.

7. Since event e always is followed by exactly one of the possible events e', e'', $\ldots$, the normalized or absolute probabilities must add up to unity. The probability that event e' will follow event e in time t is therefore the relative probability $p(e', e, t)$ divided by the sum of all the relative probabilities:
$$P(e', e, t) = \frac{p(e', e, t)}{p(e', e, t) + p(e'', e, t) + p(e''', e, t) + \ldots}. \tag{2.19}$$
Similarly, the probability that event e'' will follow event e in time t is the relative probability $p(e'', e, t)$ divided by the sum of all the relative probabilities:
$$P(e'', e, t) = \frac{p(e'', e, t)}{p(e', e, t) + p(e'', e, t) + p(e''', e, t) + \ldots}. \tag{2.20}$$

2.6 2-vectors

For our purposes, a **2-vector** is an ordered pair (x, y) of real numbers, first x and then y. One may use 2-vectors to locate addresses in a well-labeled city. In New York, for example, the 2-vector $(50, 5)$ might label the intersection of 50th Street and 5th Avenue, a very expensive place. In **Cartesian coordinates**, the 2-vector (x, y) labels the point that is x units to the right of the y-axis and y units above the x-axis. I have placed seven 2-vectors in Fig. 2.3 approximately at the points they represent. The x-axis is in blue, and y-axis in red.

One can add 2-vectors. The sum of the 2-vectors (x, y) and $u, v)$ is the 2-vector $(x + u, y + v)$. One also can multiply 2-vectors by real numbers. For instance, $3(x, y) = (3x, 3y)$.

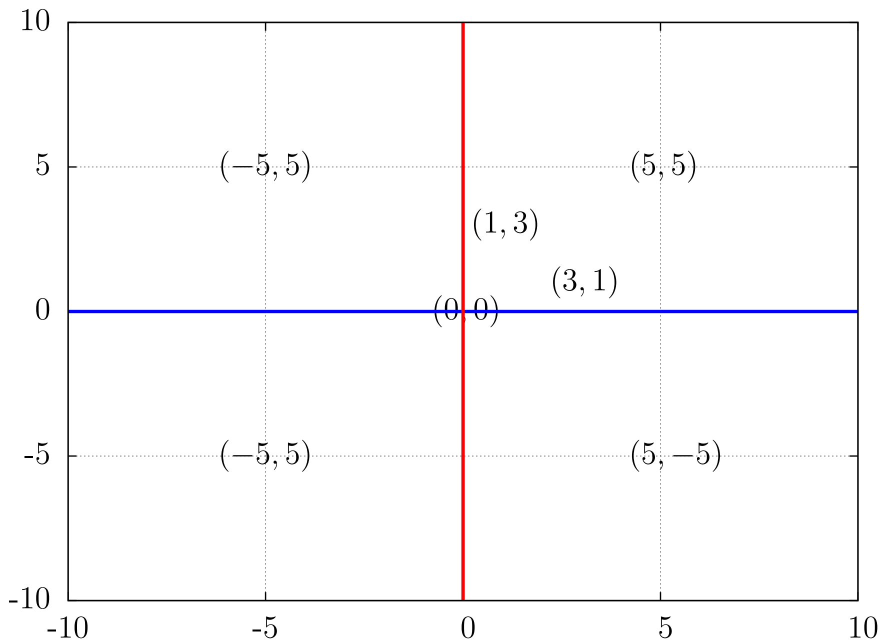


Figure 2.3: Here are seven 2-vectors placed approximately at the points they label. The point $(0, 0)$ is called the **origin** of the coordinate system. The red line is the y-axis, and the blue one is the x-axis.

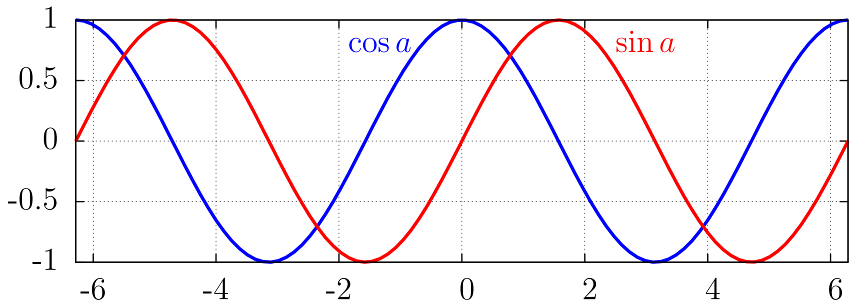


Figure 2.4: The functions $\cos a$ (blue) and $\sin a$ (red) are plotted from $a = -2\pi$ to $a = 2\pi$ radians. These functions are **periodic** with period 2π, that is, $\cos(a \pm 2\pi) = \cos a$ and $\sin(a \pm 2\pi) = \sin a$. They change sign when the angle changes by π, that is $\cos(a + \pi) = -\cos a$ and $\sin(a + \pi) = -\sin a$.

2.7 Sine and Cosine

An angle a is measured either in degrees or in radians, an angle of 360 degrees being $2\pi = 6.28$ radians. Figure 2.4 plots the trigonometric functions $\cos a$ and $\sin a$ as the angle a goes from $a = -2\pi$ to $a = 2\pi$ radians. These functions are **periodic** with period 2π, that is,

$$\begin{aligned} \cos(a \pm 2\pi) &= \cos a \\ \sin(a \pm 2\pi) &= \sin a. \end{aligned} \tag{2.21}$$

They change sign whenever the angle a is increased by $\pi = 3.14$

$$\begin{aligned} \cos(a + \pi) &= -\cos a \\ \sin(a + \pi) &= -\sin a \end{aligned} \tag{2.22}$$

as shown in the figure. For any angle a, the sum of the squares of these two trig functions is unity

$$\cos^2 a + \sin^2 a = 1. \tag{2.23}$$

2.8 Amplitudes and Probabilities Again

In quantum mechanics, the amplitude A that event e' will happen at a time t after event e is a 2-vector

$$A = (x, y) \tag{2.24}$$

in which the numbers x and y depend upon e, e', and t. The probability P that event e' will happen at a time t after event e is

$$P = x^2 + y^2. \tag{2.25}$$

Quantum mechanics gives rules for computing the amplitude $A = (x, y)$ for a given sequence of events: $e \to e'$ in time t. The rule is this:

1. Make a list of all the ways in which the physical system can go from event e to event e' in time t.

2. Compute the **action**s S_1, S_2, S_3, ... of all of these processes that go from event e to event e' in time t.

3. Add them up: apart from an overall normalization factor, the components of the amplitude $A = (x, y)$ are

$$\begin{aligned} x &= \cos(2\pi S_1/h) + \cos(2\pi S_2/h) + \cos(2\pi S_3/h) + \ldots \\ y &= \sin(2\pi S_1/h) + \sin(2\pi S_2/h) + \sin(2\pi S_3/h) + \ldots \end{aligned} \tag{2.26}$$

in which $\cos a$ and $\sin a$ are the trigonometric sine and cosine functions for an angle a measured in radians (2π radians is 360 degrees). Figure 2.4 plots the functions $\cos a$ and $\sin a$ from $a = -2\pi$ to $a = 2\pi$ radians. Keep in mind that these functions are **periodic**, that is, $\cos(a \pm 2\pi) = \cos a$ and $\sin(a \pm 2\pi) = \sin a$ and that they change sign when π is added to the angle a.

Usually, there are many processes that lead from e to e' in time t, and each has a different action S. The physics is much simpler when there effectively are only two processes that go from event e to event e'. In this case, our formulas (2.26) for the components of the amplitude $A = (x, y)$ reduce to

$$\begin{aligned} x &= \cos(2\pi S_1/h) + \cos(2\pi S_2/h) \\ y &= \sin(2\pi S_1/h) + \sin(2\pi S_2/h) \end{aligned} \tag{2.27}$$

apart from a normalization factor.

As a first example, we assume that the actions of the two processes differ by half of Planck's constant h so that $S_2 = S_1 + h/2$. Now the phase $2\pi S_2/h$ of the second terms in (2.27) is the same as that of the first terms apart from π

$$\frac{2\pi S_2}{h} = \frac{2\pi(S_1 + h/2)}{h} = \frac{2\pi S_1}{h} + \pi. \tag{2.28}$$

But we've seen (2.22) that the sine and cosine change sign when the angle a is increased by π. It follows that the first and second terms in (2.27) cancel

$$\begin{aligned} x &= \cos(2\pi S_1/h) - \cos(2\pi S_1/h) = 0 \\ y &= \sin(2\pi S_1/h) - \sin(2\pi S_1/h) = 0. \end{aligned} \tag{2.29}$$

Thus the amplitude $A = (x, y)$ vanishes. Rule (2.25) of quantum mechanics says that the probability P is the sum of the squares of the two components of the amplitude $A = (x, y)$, so P in this case is zero. The probability that event e' will follow event e in time t is $P = 0$. This cancellation is an example of **destructive interference**.

In the second example, we assume that the actions of the only two processes that go from event e to event e' differ by Planck's constant h so that $S_2 = S_1 + h$. Now our formulas for the components of the amplitude $A = (x, y)$ become

$$\begin{aligned} x &= \cos(2\pi S_1/h) + \cos(2\pi S_2/h) \\ y &= \sin(2\pi S_1/h) + \sin(2\pi S_2/h) \end{aligned} \tag{2.30}$$

apart from a normalization factor. Notice that the phase $2\pi S_2/h$ of the second terms is the same as that of the first terms apart from 2π

$$\frac{2\pi S_2}{h} = \frac{2\pi(S_1 + h)}{h} = \frac{2\pi S_1}{h} + 2\pi. \tag{2.31}$$

Thus, since by (2.21) $\cos a$ and $\sin a$ are periodic functions of the angle a with period 2π, it follows that $\cos(2\pi S_2/h) = \cos(2\pi S_1/h)$ and $\sin(2\pi S_2/h) = \sin(2\pi S_1/h)$. The first and second terms of x and y are equal, and their sums are

$$\begin{aligned} x &= 2\cos(2\pi S_1/h) \\ y &= 2\sin(2\pi S_1/h) \end{aligned} \tag{2.32}$$

apart from a normalization factor, which in this case is in effect $N = 1/2$. Rule (2.25) of quantum mechanics says that the probability P is the sum of the squares of the two components of the amplitude $A = (x, y)/2$, so P in this case is

$$\begin{aligned} P &= \frac{1}{4}\left(x^2 + y^2\right) = \frac{1}{4}\left[(2\cos(2\pi S_1/h))^2 + (2\sin(2\pi S_1/h))^2\right] \\ &= \cos^2(2\pi S_1/h) + \sin^2(2\pi S_1/h) = 1. \end{aligned} \tag{2.33}$$

Event e' follows event e in time t with certainty. This case is an example of **constructive interference**.

Figure 2.5: The laser beam hit two slits 0.4 mm wide, 0.125 mm apart, and 2 m from a white screen.

2.9 Demonstration

In 1803 Thomas Young published a description of a two-slit experiment that illustrates both constructive and destructive interference. I redid in class his experiment using a laser that emitted photons whose wave-length was $\lambda = 633$ nm $= 633 \times 10^{-9}$ m. The laser beam hit two slits that were 0.4 mm wide and were 0.125 mm apart. The slits were 2 m from a white screen as in Fig. 2.6. The image on the screen is shown in Fig. 2.6.

Event e is that a photon leaves the front of the laser at time $t = 0$; event e' is that the photon hits the screen at a point that is a distance d to the left or right of central maximum at time t. There are essentially two possible processes: either the photon goes through the left slit with action S_1 or it goes through the right slit with action S_2. The difference of the two actions depends on the difference in the lengths of the two paths.

The geometry is illustrated by Fig. 2.7, which is not to scale. The baffle with the two slits is the horizontal line $y = 0$ near the bottom of the figure. The screen lighted by the laser beam is the horizontal line $y = 10$ near the top of the figure. The angle between the vertical blue line and the red line S_1 is the same as the angle between the black line of the baffle and the pink line. Since these angles are equal and very tiny, the tangent of the first is approximately the sine of the second, $d/L \approx \ell/s$. So the difference in the

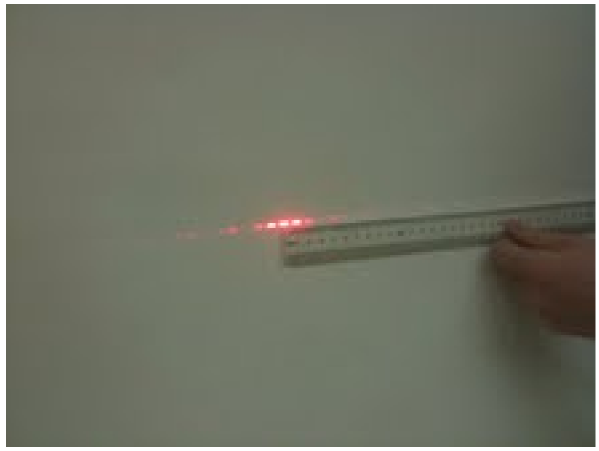


Figure 2.6: There is a central maximum separated from two secondary maxima by black fringes. The distance between the central and secondary maxima was about 1 cm.

two path lengths is approximately

$$\ell = sd/L. \tag{2.34}$$

By our rule of thumb (2.14), the action of the photon that follows the path through the right slit is

$$S_2 = h\left(\frac{L_2}{\lambda} - \nu t\right) \tag{2.35}$$

while that of the one through the left slit is

$$S_1 = h\left(\frac{L_1}{\lambda} - \nu t\right). \tag{2.36}$$

Thus since $L_1 = L_2 + \ell$, the action S_1 is

$$S_1 = h\left(\frac{L_2 + \ell}{\lambda} - \nu t\right) = S_2 + h\frac{\ell}{\lambda}. \tag{2.37}$$

By (2.34), the difference ℓ in the lengths of the paths is $\ell = sd/L$, and so the actions differ by

$$S_1 = S_2 + h\frac{\ell}{\lambda} = S_2 + h\frac{sd}{\lambda L}. \tag{2.38}$$

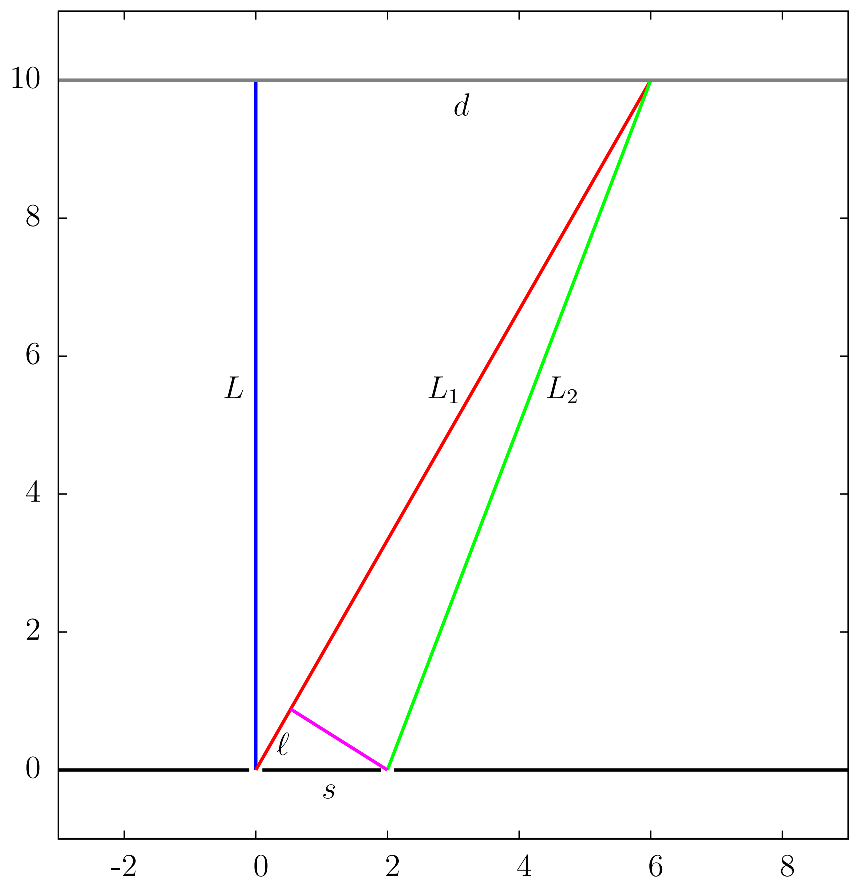


Figure 2.7: Geometry of the double-slit demo: The first slit is where the blue and red lines meet; the second slit is where the pink line meets the green one. The angle between the vertical blue line and the red line L_1 is the same as the angle between the black line of the baffle and the pink line. Since these angles are equal and very tiny, the tangent of the first is approximately the sine of the second, $d/L \approx \ell/s$.

We expect destructive interference when the actions differ by half of Planck's constant, that is, when

$$\frac{sd}{\lambda L} = \frac{1}{2}. \tag{2.39}$$

So the first dark fringe should be a distance $d = \lambda L/(2s)$ to the right (or left) of the central maximum. In the in-class demo, the wave-length of the laser was $\lambda = 633$ nm or $\lambda = 633 \times 10^{-9}$ m, the separation s between the two slits was $s = 0.125$ mm or $s = 0.125 \times 10^{-3}$ m, and the distance from the two slits to the screen was 2 m. The distance to the first dark fringe then should be

$$d = 633 \times 10^{-9} \times 2/(2 \times 0.125 \times 10^{-3})\,\mathrm{m} = 0.005\,\mathrm{m} = 5\,\mathrm{mm}. \tag{2.40}$$

This is what we saw in class, and what we dimly can see in Fig. 2.6.

We expect constructive interference when the actions differ by Planck's constant, that is, when

$$\frac{sd}{\lambda L} = 1. \tag{2.41}$$

The center of the first secondary maximum should then be a distance

$$d = \frac{\lambda L}{s} = 633 \times 10^{-9} \times 2/(0.125 \times 10^{-3})\,\mathrm{m} = 0.01\,\mathrm{m} = 1\,\mathrm{cm} \tag{2.42}$$

which is what we saw in class, and what we dimly can see in Fig. 2.6.

2.10 Complex Numbers

You can think of complex numbers as 2-vectors. In fact, that is how they are written in computer languages such as Fortran. But you also can multiply two complex numbers together. To do so, we introduce a new number i whose square is -1:

$$i \times i = i^2 = -1. \tag{2.43}$$

Next we write the general 2-vector (x, y) as $x + iy$. Then we just multiply and replace i^2 by -1. Thus, the product of the two 2-vectors (x, y) and (u, v) thought of as the complex numbers $x + iy$ and $u + iv$ is

$$(x+iy)\times(u+iv) \equiv (x+iy)(u+iv) = xu+ixv+iyu+i^2yv = xu-yv+i(xv+yu). \tag{2.44}$$

You can find a gentle and interesting introduction to complex numbers by googling "NY Times complex numbers 2010/03/07."

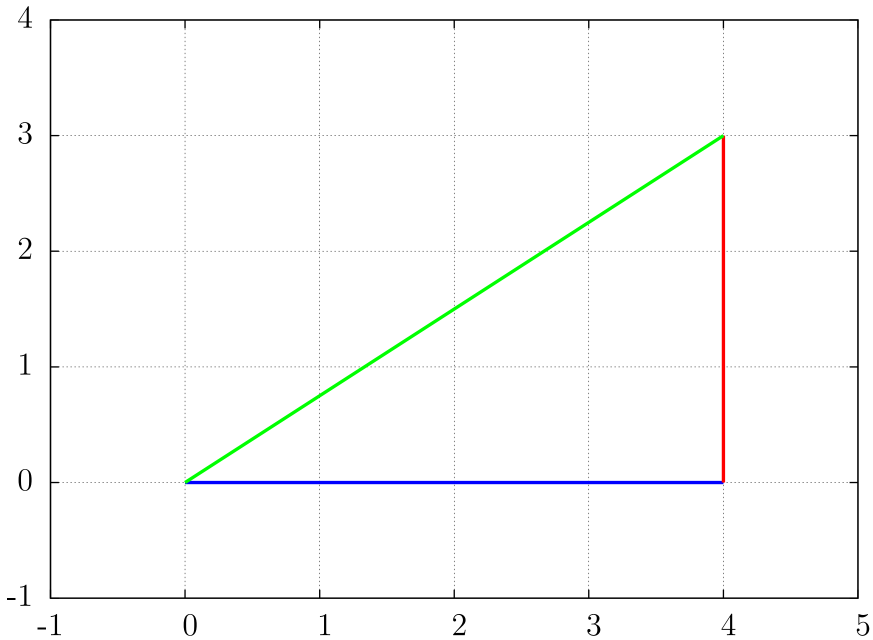


Figure 2.8: The 2-vector $(4, 3)$ or its equivalent the complex number $4 + 3i$ is the point where the green hypotenuse touches the red vertical line. The length of the green hypotenuse is 5.

2.11 Complex Numbers of Unit Length

The length ℓ of the 2-vector (x, y) or of the equivalent complex number $x+iy$ is the square-root

$$\ell = \sqrt{x^2 + y^2}. \tag{2.45}$$

Fig. 2.8 shows the case of $x = 4$ and $y = 3$ in which the length ℓ of the complex number $4 + 3i$ is the length of the hypotenuse

$$\ell = \sqrt{4^2 + 3^2} = \sqrt{25} = 5. \tag{2.46}$$

Some complex numbers $x + iy$ are of unit length; they have length $\ell = \sqrt{x^2 + y^2} = 1$. For any angle a, the sum of $\cos^2 a$ plus $\sin^2 a$ is unity. So every 2-vector $(\cos a, \sin a)$ or equivalently every complex number $\cos a + i \sin a$ has length 1. They are the points on a circle of radius 1 and center at $(0, 0)$, which is called the unit circle.

Chapter 3

Relativity

The media confuse Einstein's relativity with vague sentiments to the effect that truth is relative, that anything goes. Actually, relativity is about sameness and constancy. The essential idea of **general relativity** is that the equations of physics look the same in all coordinate systems. The basic idea of **special relativity** is that physics looks the same in any coordinate system that is moving at a constant velocity relative to the cosmic background microwave radiation. We see evidence of this in daily life. On an airplane, we don't need to know the velocity of the plane before pouring soda into a glass of ice.

Now imagine two coordinate systems, one (x', y', z', t') moving in the x-direction relative to the other at speed v and the other (x, y, z, t) "stationary." Since the relative motion is in the x-direction, we'll take it for granted that $y' = y$ and $z' = z$. Take the moving coordinate system to be an open railroad car with an x' axis down its center-line parallel to the train tracks assumed to be straight. The car has a clock measuring time as t'. We'll assume that the origins of the two coordinate systems coincide in the sense that $x' = x = 0$ at $t' = t = 0$.

The simplest way to relate (x', t') to (x, t) is by means of linear relations involving four parameters a, b, f, and g:

$$\begin{aligned} x' &= ax + bt \\ t' &= fx + gt. \end{aligned} \tag{3.1}$$

Here I'm measuring distance in light seconds or light years so that x and t have the same dimensions.

As shown chapter 5, one can derive the values of the parameters a, b, f, and g just by assuming that the speed of light is the same in both coordinate systems. This derivation is easier, however, if we assume that physics is the same in both coordinate systems to the extent that

$$t'^2 - x'^2 = t^2 - x^2. \tag{3.2}$$

Expressing x' and t' in terms of x and$t$ by using (3.1), we have

$$(fx + gt)^2 - (ax + bt)^2 = t^2 - x^2 \tag{3.3}$$

or

$$(g^2 - b^2)t^2 + 2(fg - ab)xt - (a^2 - f^2)x^2 = t^2 - x^2 \tag{3.4}$$

which must hold for all x and t. It follows that

$$\begin{aligned} g^2 - b^2 &= 1 \\ fg - ab &= 0 \\ a^2 - f^2 &= 1. \end{aligned} \tag{3.5}$$

We find $f^2 = a^2 - 1$ and $g^2 = b^2 + 1$ and then to solve the middle equation in the form $f^2 g^2 = a^2 b^2$:

$$(a^2 - 1)(b^2 + 1) = a^2\, b^2 \tag{3.6}$$

which leaves us with $b^2 = a^2 - 1$. So we have formulas for b, f, and g in terms of a or b. Actually, b is the better choice

$$\begin{aligned} a &= \sqrt{b^2 + 1} \\ f &= b \\ g &= \sqrt{b^2 + 1} \end{aligned} \tag{3.7}$$

Our equations (3.1) relating x' and t' to x and t then are

$$\begin{aligned} x' &= \sqrt{b^2 + 1}\, x + b\, t \\ t' &= b\, x + \sqrt{b^2 + 1}\, t. \end{aligned} \tag{3.8}$$

Let's look at these equations (3.8) for small v, where we know what they should be. If the (x', t') coordinate system is moving in the x direction, and if v is tiny, then we expect

$$\begin{aligned} x' &\approx x - v\, t \\ t' &\approx t. \end{aligned} \tag{3.9}$$

If we now go back to ordinary units, putting back the speed of light c into equations (3.8), then we get

$$
\begin{aligned}
x' &= \sqrt{b^2+1}\,x + b\,c\,t & (3.10)\\
t' &= b\,x/c + \sqrt{b^2+1}\,t. & (3.11)
\end{aligned}
$$

By comparing these equations with the ones (3.9) we know for tiny v, we find that if the relative speed v is slow, then

$$b \approx -\frac{v}{c}. \tag{3.12}$$

But what if the speed v is big? As we'll see in a moment, the exact answer is

$$b = -\frac{v}{c\sqrt{1 - v^2/c^2}} \tag{3.13}$$

in which v is the speed of the (x', t') coordinate system in the x-direction.

The other quantity that appears in (3.8) is

$$\sqrt{1+b^2} = \sqrt{1 + v^2/(c^2 - v^2)} = \sqrt{c^2/(c^2 - v^2)} = 1/\sqrt{1 - v^2/c^2}. \tag{3.14}$$

So our final equations (3.8) are

$$
\begin{aligned}
x' &= (x - vt)/\sqrt{1 - v^2/c^2}\\
t' &= (t - xv/c^2)/\sqrt{1 - v^2/c^2} & (3.15)
\end{aligned}
$$

which is called a **Lorentz transformation**. Physicists often use the Greek letter gamma for the inverse square-root

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{3.16}$$

In this notation, the Lorentz transformation (3.15) is

$$
\begin{aligned}
x' &= \gamma\,(x - vt)\\
t' &= \gamma\,(t - xv/c^2). & (3.17)
\end{aligned}
$$

The coefficients of x and t are functions of the ratio v/c.

Let's now consider an example sketched in Fig. 3.1 that illustrates how moving clocks slow down and shows that b is given by equation (3.13). Suppose a person in the open railroad car measures the time t' that it takes for

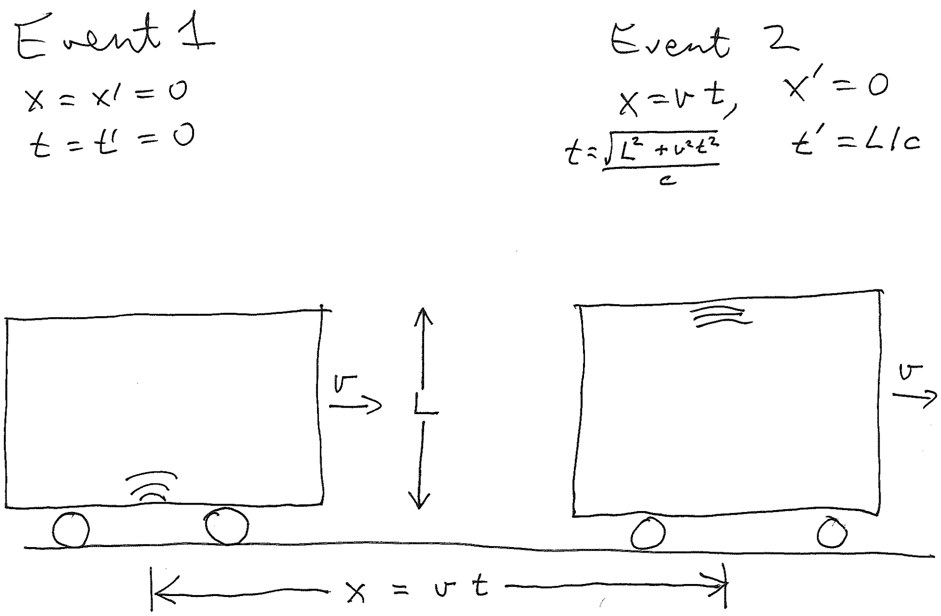


Figure 3.1: A light pulse rises a distance L in the frame of the railroad car moving at speed v. Seen from the ground, the pulse goes to the right by vt while rising a distance L for a total distance of $\sqrt{L^2 + (vt)^2}$.

a vertical light pulse to travel a distance L from the bottom of the car to the top of the car when the car is moving at speed v in the x-direction. In the coordinate system of the moving railroad car, the first event is the emission of the light pulse from the origin of the moving coordinate system at $x' = 0$ and $t' = 0$, and the second event is the arrival of the light pulse at the top of the car at $x' = 0$ and $ct' = L$:

$$x' = 0 \quad \text{and} \quad ct' = L \quad \text{is second event.} \tag{3.18}$$

Here we assume that the speed of light is c in both the moving and the stationary coordinate systems.

To make things simple, we have the origin of the two coordinate systems the same, that is, $x' = 0$ and $t' = 0$ represent the same event as $x = 0$ and $t = 0$. Then in the stationary coordinate system, the first event is the emission of the light pulse at $x = 0$ and $t = 0$, and the second event is its arrival at the top of the car at time t by which time the top of the car will be at $x = vt$. So in the stationary coordinate system, the distance the pulse moves in time t is $\sqrt{L^2 + v^2t^2}$. Since the speed of light must be the same in both systems, we must have $ct = \sqrt{L^2 + v^2t^2}$. But we just saw in (3.18) that the value of t' for the second event satisfies $ct' = L$, so we have

$$ct = \sqrt{(ct')^2 + (vt)^2}. \tag{3.19}$$

Squaring both sides, we get

$$(ct)^2 = (ct')^2 + (vt)^2 \tag{3.20}$$

which implies that $t'^2 = (1 - v^2/c^2)t^2$ or

$$t' = \sqrt{1 - v^2/c^2}\ t. \tag{3.21}$$

So the time t' that the light takes to go to the top of the moving car in the moving (x', t') system is less than the time t of the same process in the still (x, t) system. Moving clocks run slow, a phenomenon known as **time dilation**. We saw this when we detected muons, made by cosmic rays some 10 km above sea level, in a spark chamber and in Geiger counters. If the clocks of the moving muons ran as fast as our laboratory clocks, then virtually all the muons would have decayed into electrons and neutrinos long before reaching Regener Hall.

Let's now use the equation (3.10) to find b, that is, to derive the exact relation (3.13). Since by (3.18) the value of x' for the second event is $x' = 0$, we have

$$x' = 0 = \sqrt{b^2 + 1}\, x + b\, c\, t. \tag{3.22}$$

But $x = vt$, so this is

$$0 = \sqrt{b^2 + 1}\, vt + b\, c\, t. \tag{3.23}$$

Canceling t, we get

$$b = -(v/c)\sqrt{b^2 + 1}. \tag{3.24}$$

Now we squaring both sides, we find

$$b^2 = (v^2/c^2)(b^2 + 1) \tag{3.25}$$

or

$$(1 - v^2/c^2)\, b^2 = v^2/c^2. \tag{3.26}$$

Isolating b^2 and taking the square-root of both sides, we get

$$b = \pm \frac{v}{c\sqrt{1 - v^2/c^2}} \tag{3.27}$$

which is the exact result (3.13). The minus sign applies when the (x', t') system is moving at speed v in the x-direction; the plus sign when the (x', t') system is moving at speed v in the negative x-direction, that is, backwards.

Chapter 4

Chemistry

The most interesting part of chemistry is the part that is relevant to biology; this is the chemistry of carbon, usually called **organic chemistry**. Cells are made from the more abundant chemical elements, mainly H, C, O, N, Ca, Mg, Na, K, P, among others. These atoms are held together in molecules by covalent and ionic bonds, although molecules bound by ionic bonds also are called *salts*.

Covalent bonds are the strongest kind of chemical bond. A good example is the bond between two hydrogen atoms. It takes 4.75 eV to separate a hydrogen molecule H_2 into two hydrogen atoms.

The cell uses sugars, fatty acids, amino acids, and nucleotides to make larger molecules such as polysaccharides, fats, lipids, proteins, and nucleic acids.

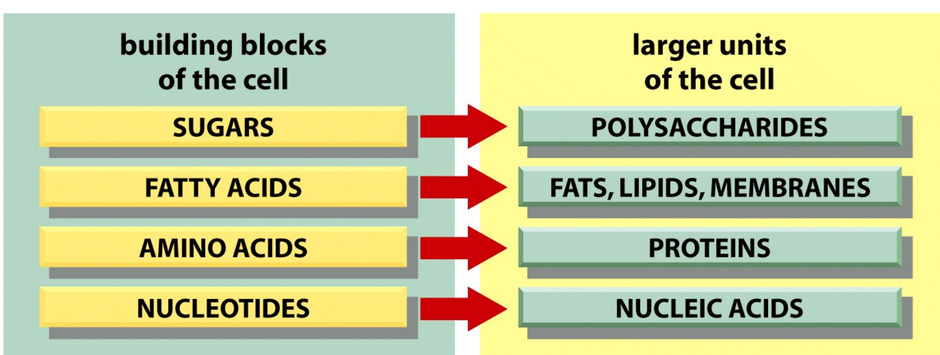
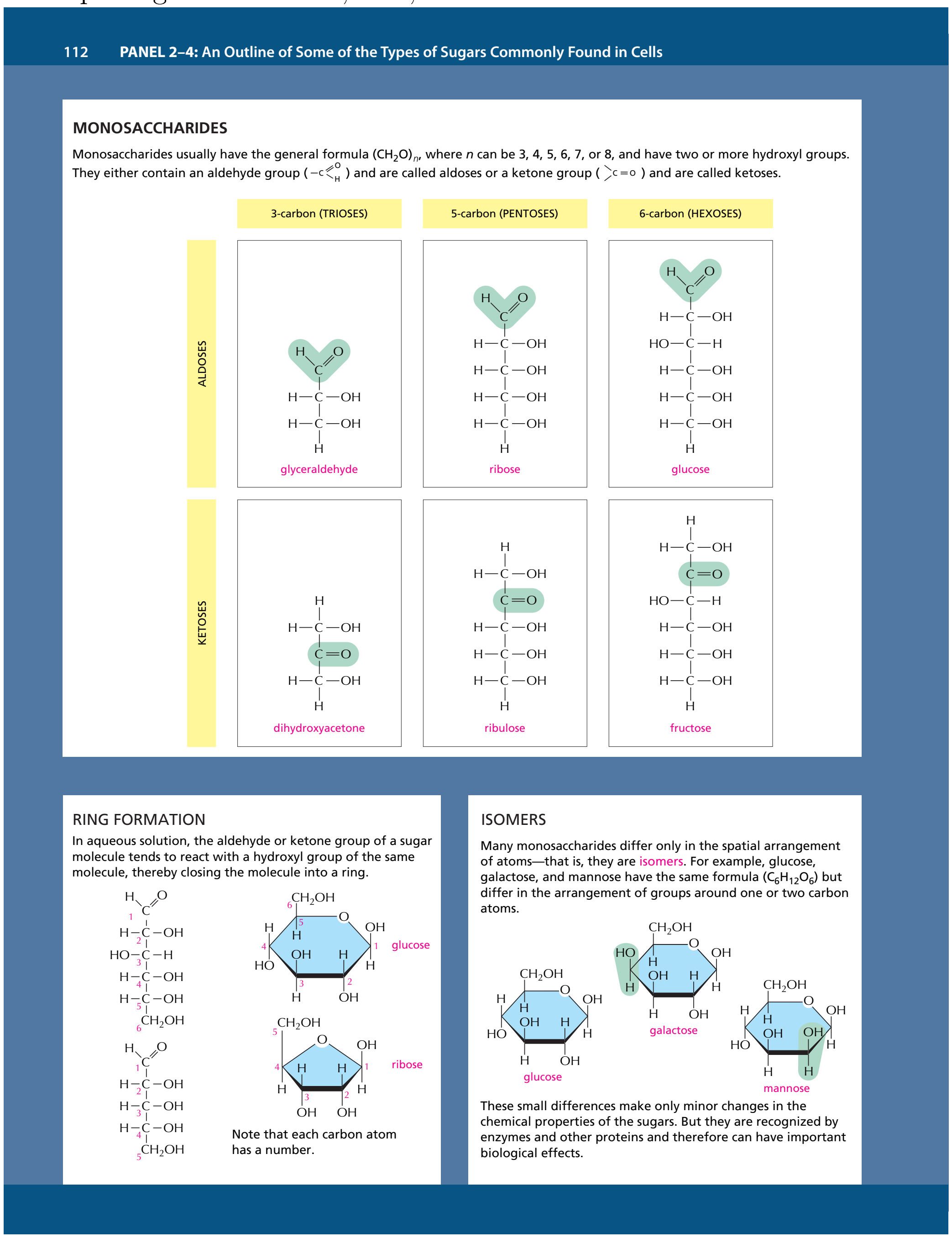


Figure 2-17 Molecular Biology of the Cell 5/e (© Garland Science 2008)

4.1 Sugars

Some simple sugars with three, four, and five carbon atoms:


112 PANEL 2–4: An Outline of Some of the Types of Sugars Commonly Found in Cells

MONOSACCHARIDES

Monosaccharides usually have the general formula $(CH_2O)_n$, where *n* can be 3, 4, 5, 6, 7, or 8, and have two or more hydroxyl groups. They either contain an aldehyde group and are called aldoses or a ketone group and are called ketoses.

RING FORMATION

In aqueous solution, the aldehyde or ketone group of a sugar molecule tends to react with a hydroxyl group of the same molecule, thereby closing the molecule into a ring.

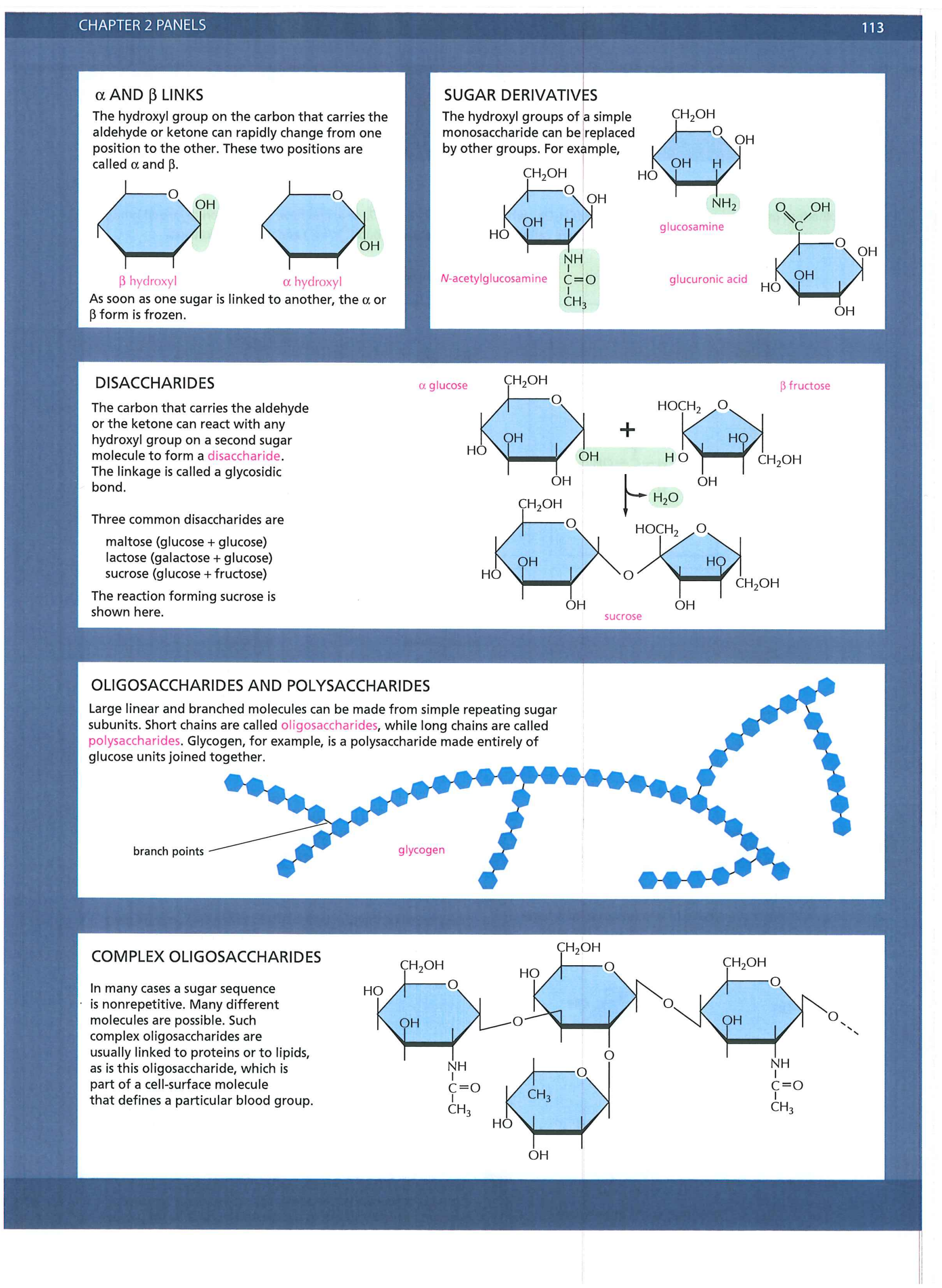
Note that each carbon atom has a number.

ISOMERS

Many monosaccharides differ only in the spatial arrangement of atoms—that is, they are isomers. For example, glucose, galactose, and mannose have the same formula ($C_6H_{12}O_6$) but differ in the arrangement of groups around one or two carbon atoms.

These small differences make only minor changes in the chemical properties of the sugars. But they are recognized by enzymes and other proteins and therefore can have important biological effects.

More complex sugars:



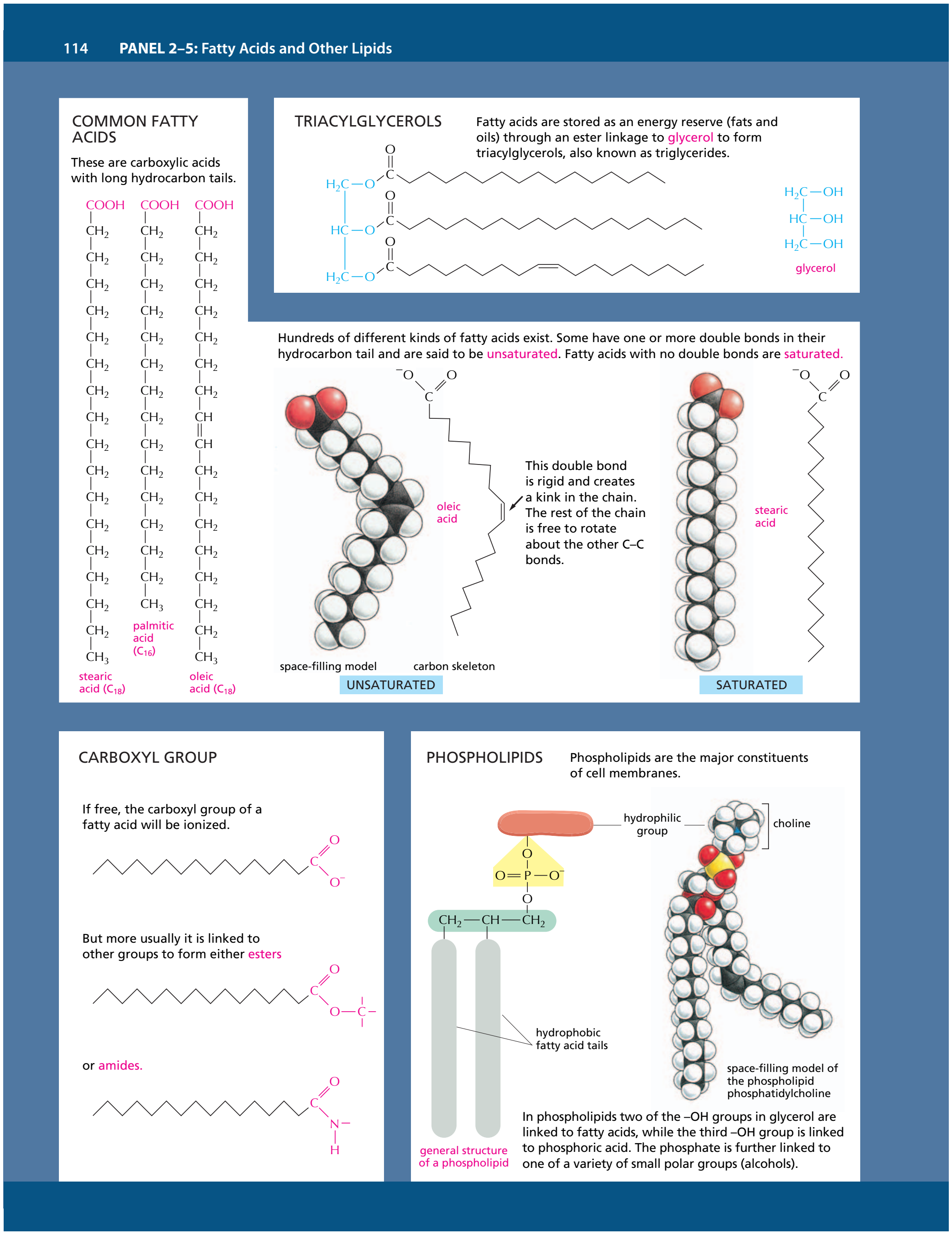
Sugars form long polymers with and without branches. Cellulose is a polysaccharide of glucose found in the cell walls of plants; it is the most abundant organic chemical on Earth. The chitin of insect exoskeletons and fungal cell walls is also a polysaccharide. Polysaccharides are the main components of slime, mucus, and gristle. Oligosaccharides covalently linked to proteins are glycoproteins; linked to lipids, they are glycolipids.

4.2 Lipids

In the jargon of biochemistry, fatty acids and fats are called **lipids**. A **fatty acid** is a hydrocarbon that ends in a carboxylic-acid group COOH as shown in the upper-left part of Fig. 4.2.

Glycerol is a chain of three carbon atoms, each attached to a hydroxyl group and saturated with hydrogen atoms; it's in the upper-right corner of Fig. 4.2.

a **fat** or a **triacylglycerol** is three fatty acids fused to the hydroxyl groups of a glycerol molecule by **ester** linkages; it's shown in the top of Fig. 4.2.



114 PANEL 2–5: Fatty Acids and Other Lipids
COMMON FATTY ACIDS
These are carboxylic acids with long hydrocarbon tails.
COOH
stearic acid (C_{18})
palmitic acid (C_{16})
oleic acid (C_{18})
TRIACYLGLYCEROLS
Fatty acids are stored as an energy reserve (fats and oils) through an ester linkage to glycerol to form triacylglycerols, also known as triglycerides.
glycerol
Hundreds of different kinds of fatty acids exist. Some have one or more double bonds in their hydrocarbon tail and are said to be unsaturated. Fatty acids with no double bonds are saturated.
oleic acid
This double bond is rigid and creates a kink in the chain. The rest of the chain is free to rotate about the other C–C bonds.
stearic acid
space-filling model
carbon skeleton
UNSATURATED
SATURATED
CARBOXYL GROUP
If free, the carboxyl group of a fatty acid will be ionized.
But more usually it is linked to other groups to form either esters
or amides.
PHOSPHOLIPIDS
Phospholipids are the major constituents of cell membranes.
hydrophilic group
choline
hydrophobic fatty acid tails
space-filling model of the phospholipid phosphatidylcholine
general structure of a phospholipid
In phospholipids two of the –OH groups in glycerol are linked to fatty acids, while the third –OH group is linked to phosphoric acid. The phosphate is further linked to one of a variety of small polar groups (alcohols).

Chapter 5

Mathematical Notes

5.1 The Lorentz Transformation

The relation between the time-and-space coordinates of two inertial coordinate systems follows from the assumption that the speed of light is the same in the two frames. Imagine two (x, y, z, t) coordinate systems, one (x', y', z', t') moving in the x-direction relative to the other at speed v and the other (x, y, z, t) "stationary." Since the relative motion is in the x-direction, we'll take it for granted that $y' = y$ and $z' = z$. Take the moving coordinate system to be a open railroad car with an x' axis down its center-line and a clock measuring time as t'. We'll assume that the origins of the two coordinate systems coincide in the sense that $x' = x = 0$ at $t' = t = 0$. That's the point $(0, 0)$ in Fig. 5.1.

A light ray emitted from the common origin at $t' = t = 0$ will go a distance $x = ct$ in time t. So it's the orange line at 45 degrees in the figure.

The origin $x' = 0$ of the open railroad car's spatial coordinate system gets to $x = vt$ at time t and is the green line at about 30 degrees in the figure. These points are labeled $(0, t')$ in the open railroad car's coordinate system; they form the positive ct' axis of that system.

The speed of light must be the same in both coordinate systems. So just as the orange line that represents the light ray makes the same angle with the x-axis as with the ct-axis, so too it must make the same angle with the x'-axis as with the ct'-axis, as in the figure.

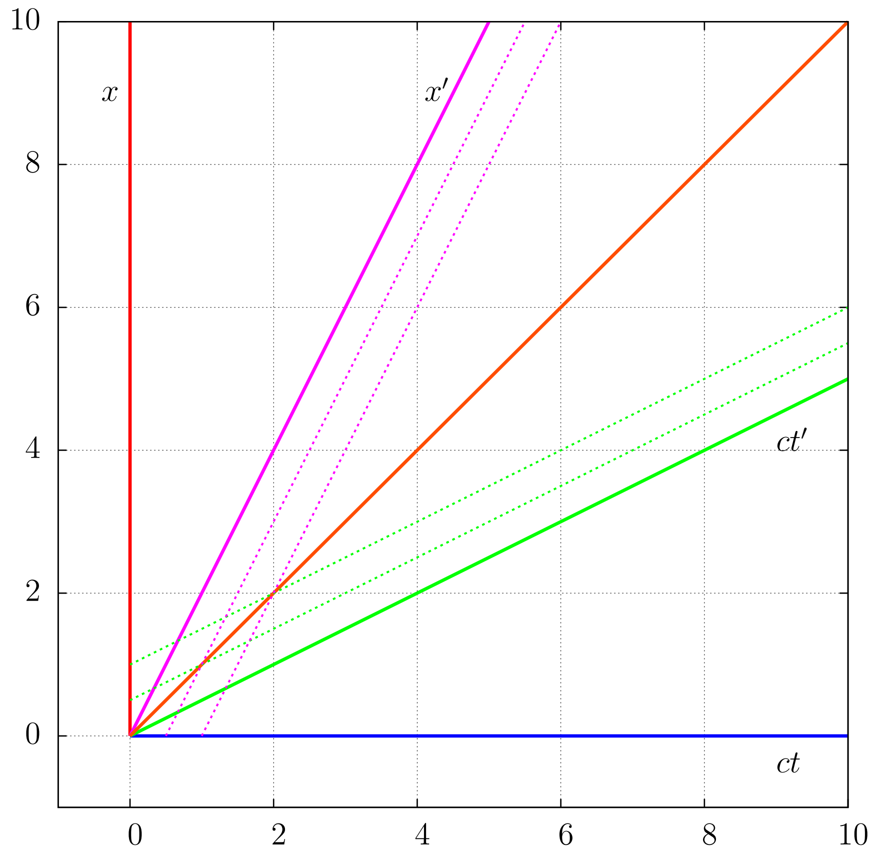


Figure 5.1: The orange line at 45 degrees is a light ray emitted at the common origin $(x', t') = (x, t)$ of the two coordinate systems. The (x', t') coordinate system is moving at speed v in the x-direction. The events $x' = 0$ and $t' \geq 0$ are the center of the open railroad car at and after takeoff; these points form the green line labeled ct'. The events $ct' = 0$ and $x' \geq 0$ are the x'-axis of the open railroad car at takeoff, $t' = 0$; these points form the pink line labeled x'. The grid lines of the coordinate system of the open railroad car are the dotted pink and green lines. They respectively are parallel to the x' and ct' axes. The light pulse will travel at the same speed of light, c, if the pink x'-axis makes the same angle with the orange light ray as does the green ct' axis.

Now let's derive the linear relations

$$x' = ax + bt \tag{5.1}$$
$$t' = fx + gt \tag{5.2}$$

that give x' and t' in terms of x and t for the case in which the open railroad car moves at speed v in the x-direction of the stationary system. We'll again measure distance in light seconds so that x and t (and x' and t') have the same units.

First, we note that the point $x' = 0$ moves with speed v in the stationary system, so by (5.1) we have

$$0 = avt + bt. \tag{5.3}$$

Thus, canceling t, we find that

$$b = -av. \tag{5.4}$$

Next we imagine a light pulse emitted from the common origin $x' = t' = x = t = 0$. Because the speed of light is the same in both systems, the light ray follows the lines $x = t$ and $x' = t'$. So the constancy of the speed of light together with the linear relations (5.1 & 5.2 & 5.4) give us

$$t' = at - avt = (a - av)t \tag{5.5}$$
$$t' = ft + gt = (f + g)t. \tag{5.6}$$

Thus canceling t, we have

$$a(1 - v) = f + g. \tag{5.7}$$

Now the linear relations (5.1 & 5.2) that give x' and t' in terms of x and t imply that x and t are given by

$$x = (gx' - bt')/(ag - bf) \tag{5.8}$$
$$t = (-fx' + at')/(ag - bf). \tag{5.9}$$

(One may verify these relations explicitly by using (5.1 & 5.2) to express x' and t' in terms of x and t and check that one just gets $x = x$ and $t = t$.)

Now the symmetry of the two coordinate systems means that just as the point $x' = 0$ moves at speed v in the x-direction in the stationary system, so too the point $x = 0$ moves at speed $-v$ in the x'-direction in the open railroad car's system. That is, the point $x = 0$ corresponds to $x' = -vt'$ in that system. But then equation (5.8) gives us for $x = 0$ the relation

$$x = 0 = (g(-vt') - bt')/(ag - bf). \tag{5.10}$$

Multiplying by $(ag - bf)/t'$, and recalling (5.4) that $b = -av$, we have $-gv + av = 0$ or

$$g = a. \tag{5.11}$$

Now by substituting this value of g into (5.7), we get $a(1 - v) = f + a$ or

$$f = -av. \tag{5.12}$$

So now we have expressed all four parameters of the linear relations (5.1–5.2) in terms of the single parameter a:

$$\begin{aligned} x' &= a(x - vt) && (5.13)\\ t' &= a(t - vx). && (5.14) \end{aligned}$$

To find this last parameter a, we got back to our analysis of the light pulse that rises vertically a distance L in the frame of the open railroad car but that goes a distance $\sqrt{L^2 + v^2t^2}$ in the stationary frame. Since the speed of light is the same in both frames, we have $t' = L$ and $t = \sqrt{L^2 + v^2t^2}$, which implies that $t^2 = L^2 + v^2t^2$ or

$$t = L/\sqrt{1 - v^2}. \tag{5.15}$$

But then by equation (5.14), we have

$$t' = L = a(t - vx). \tag{5.16}$$

We now substitute (5.16) for t and $x = vt$ for x in this equation (5.16) getting

$$L = a(L/\sqrt{1 - v^2} - v^2L/\sqrt{1 - v^2}) = aL(1 - v^2)/\sqrt{1 - v^2} = aL\sqrt{1 - v^2} \tag{5.17}$$

or

$$a = \frac{1}{\sqrt{1 - v^2}}. \tag{5.18}$$

Going back to ordinary SI units, we recover the Lorentz transformation

$$\begin{aligned} x' &= \gamma\,(x - vt) \\ t' &= \gamma\,(t - xv/c^2) \end{aligned} \tag{5.19}$$

in which

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \tag{5.20}$$

So indeed the Lorentz transformation follows directly from the assumption of the constancy of the speed of light (and linearity and $y' = y$, $z' = z$ for relative motion along the x-axis).

Bibliography

[1] Steven Weinberg. *The First Three Minutes*. Basic Books, New York City, 1988.