Final Examination for Physics 102

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(Dated: May 9, 2005)

Relax. This test is supposed to be a learning experience. It will be graded generously. It is a multiple-choice test. You get several points just for printing your name on the form in the tedious way required by the machine-readable form.

Energy and Momentum

Energy, momentum, and angular momentum are all absolutely conserved. As Feynman wrote, there are many forms of energy. What is conserved is the sum of all the different forms of energy. The gravitational potential energy V of an object of mass m at a height h in a field with a gravitational acceleration g is $V = mgh$. The kinetic energy of a mass m moving at speed v is $K = (m/2)\, v^2$ as long as v is much less than the speed of light c. The momentum $\vec{p}$ of a mass m moving at speed v is $\vec{p} = m\vec{v}$. If the mass is moving very fast, then its momentum is $\vec{p} = m\vec{v}/\sqrt{1-(v/c)^2}$, and its rest energy plus its kinetic energy is $E = mv^2/\sqrt{1-(v/c)^2}$.

Question 1

A mass m at a height h in a field with a gravitational acceleration g has gravitational potential energy $V = mgh$. The mass falls from its height h to the ground. When it hits the ground, its height will be zero, and its gravitational potential energy will be zero. What will be its kinetic energy as it hits the ground? (a) Zero, (b) mgh, (c) 3, (d) $-mgh$, (e) -3.

Question 2

Imagine two sticky globs of stuff all by themselves in empty space. Glob 1 has momentum $\vec{p}_1$, and glob 2 has momentum $\vec{p}_2$. The two globs collide and stick together in one big glob. What is the momentum of the big glob? (a) Zero, (b) 2, (c) $\vec{p}_1 + \vec{p}_2$, (d) 3, (e) 4.

Question 3

Same as question 2, but now $\vec{p}_1 = (2, 0, 0)$ and $\vec{p}_2 = (0, 3, 0)$ in units of kilogram-meters-per-second. What is the momentum of the big glob? (a) (0,0,0), (b) (2,0,0), (c) (0,3,0), (d) (2,3,0), (e) 4.

Conserved Charges

Like energy, momentum, and angular momentum, the quantities electric charge, quark number, and lepton number are absolutely conserved, as far as we know. The proton has electric charge 1 in atomic units, and the electron has electric charge -1. The electron, the muon, and the tau are leptons with electric charge -1, quark number 0, and lepton number 1. Their antiparticles carry electric charge 1, quark number 0, and lepton number -1. There are three neutrinos; each has electric charge 0, quark number 0, and lepton number 1. Their antiparticles, called anti-neutrinos, have electric charge 0, quark number 0, and lepton number -1. There are six quarks (each of which carries one of three "colors"). The u, c, and t quarks carry electric charge 2/3, quark number 1, and lepton number 0; their anti-quarks carry electric charge $-2/3$, quark number -1, and lepton number 0. The d, s, and b quarks carry electric charge $-1/3$, quark number 1, and lepton number 0; their anti-quarks carry electric charge 1/3 and quark number -1, and lepton number 0. The neutron has charge 0; it is neutral. The proton and the neutron each contain 3 quarks and no leptons and so have quark number 3 and lepton number 0. They contain no leptons and have lepton number 0. Their antiparticles contain 3 anti-quarks and so carry quark number -3 and lepton number 0. These particles all are fermions with spin one-half. The photon is a boson of spin one, charge zero, quark number zero, and lepton number 0. The W^+ boson is a boson of spin one, charge 1, quark number zero, and lepton number 0; its antiparticle, the W^- boson, is a boson of spin one, charge -1, quark number zero, and lepton number 0. The Z boson is a boson of spin one, charge 0, quark number zero, and lepton number 0.

Question 4

Which of these processes conserves electric charge? (a) u quark becomes d quark + W^+, (b) u quark becomes t quark + W^+, (c) neutrino becomes proton, (d) proton becomes neutrino, (e) u quark + t quark become photon.

Question 5

Which of these processes conserves quark number? (a) u quark becomes d quark + W^+, (b) u quark + t quark become W^+, (c) electron becomes proton, (d) proton becomes neutrino, (e) u quark + t quark become W^+ + photon.

Question 6

Which of these processes conserves lepton number? (a) muon becomes electron + Z, (b) u quark + t quark become electron + muon (c) electron becomes proton, (d) proton becomes neutrino, (e) u quark + t quark become W^+ + electron.

Quantum Mechanics

Feynman summarized the essence of quantum mechanics somewhat like this:

1. If the amplitude of an event is A, then the probability P of that result is the square of the absolute value of A, that is $P = |A|^2$.

2. If a given event can happen in two different ways, with amplitudes A_1 and A_2, then the amplitude of the event is the sum of the two amplitudes, $A = A_1 + A_2$. In this case, the probability of the event is $P = |A|^2 = |A_1 + A_2|^2$. There is interference.

3. If an experiment is done to find out which way the event happens, then the interference disappears and the probability is $P = P_1 + P_2 = |A_1|^2 + |A_2|^2$.

The *amplitude* may be thought of as the amplitude of the wave that guides the particle.

Example: Suppose the amplitude A is the complex number $A = x + iy$. Then the probability is $P = |A|^2 = |x + iy|^2 = x^2 + y^2$. (Here i is the square-root of minus one, that is, $i^2 = -1$, but we won't need to use that.)

Example: Not all amplitudes are complex; some are real. Thus if the amplitude A is the real number $A = x$. Then the probability is $P = |A|^2 = |x|^2 = x^2$.

Example: If an event can happen in two different ways, with amplitudes $A_1 = x_1 + iy_1$ and $A_2 = x_2 + iy_2$, then the total amplitude is the sum of the two amplitudes, $A = A_1 + A_2 = x_1 + x_2 + i(y_1 + y_2)$, and the probability is $P = |A|^2 = |A_1 + A_2|^2 = (x_1 + x_2)^2 + (y_1 + y_2)^2$. For instance, if $A_1 = 0.1 + i0.2$ and $A_2 = 0.2 - i0.1$, then the amplitude of the event is $A = A_1 + A_2$, and so in this case, the amplitude is $A = 0.3 + i0.1$. The probability then is $P = |A|^2 = |0.3 + i0.1|^2 = (0.3)^2 + (0.1)^2 = 0.09 + 0.01 = 0.1$ or 10%.

Question 7

Suppose that the amplitude A that a photon hits a detector is $A = 0.1$ or one-tenth. What is the probability P that the photon hits the detector? (a) 0.0, (b) 0.1, (c) 0.2, (d) 0.01, (e) 0.9.

Question 8

If the amplitude A that a electron hits a detector is $A = 0.1 + i0.2$, what is the probability P that the electron hits the detector? (a) 0.05, (b) 0.1, (c) 0.3, (d) 0.5, (e) 0.9.

Question 9

Suppose now, as in the two-slit demos we saw so often in class, that a photon comes out of a laser and can arrive at a point on a screen by going through either of the two slits. We make no effort to see which slit the photon uses. If the amplitude that a photon hits a given point on the screen by going through slit one is $A_1 = 0.1 + i0.2$, and the amplitude that it hits the point by going through slit two is $A_2 = 0.2 + i0.2$, then what is the probability P that it hits the point on the screen? (a) 0.0, (b) 0.10, (c) 0.25, (d) 0.3, (e) 0.45. Hint: This is called constructive interference.

Question 10

Same as question 9, but now for a different point on the screen, the two amplitudes are $A_1 = 0.1 + i0.2$, and $A_2 = -0.1 - i0.2$. What is the probability P that the photon hits this point? (a) 0.0, (b) 0.10, (c) 0.25, (d) 0.3, (e) 0.45. Hint: This is called destructive interference.

Question 11

Now we do the experiment in such a way as to know which slit the photon passes through. That is, we somehow measure the transit of each photon. Take the amplitudes to be as in question 10. What is the probability that a photon hits the given point on the screen? (a) 0.0, (b) 0.10, (c) 0.25, (d) 0.3, (e) 0.45.

Electrodynamics

A particle that carries electric charge q and that moves with velocity $\vec{v}$ in an electric field $\vec{E}$ and a magnetic field $\vec{B}$ feels a force $\vec{F}$ that is given by $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$. So the electric force is $q\vec{E}$, and the magnetic force is the cross-product $q\vec{v} \times \vec{B}$. To find the direction of this cross-product, hold your right hand flat and oriented so that your fingers point in the direction of the velocity $\vec{v}$ and so that when you flex your fingers, they point in the direction of the magnetic field $\vec{B}$. Your thumb then will point in the direction of the magnetic component of the force, which is perpendicular to both $\vec{v}$ and $\vec{B}$. I will do the gesture for you during the test if you ask me to.

Question 12

A proton carries a positive charge equal in magnitude to the negative charge of the electron. A proton is released at speed zero in a strong electric field that points toward the south. What happens to the proton after it is released? (a) Nothing. (b) The proton decays into a neutron, a positron, and a neutrino. (c) The proton races off toward the south. (d) The proton drifts toward the east. (e) The proton gradually stops spinning.

Question 13

A electron is released at speed zero in a strong magnetic field that points toward the east. What happens to the electron after it is released? (a) Nothing; it just stays there. (b) The electron decays into a neutrino and a photon. (c) The electron races off toward the north. (d) The electron drifts toward the east. (e) The electron gradually stops spinning.

Question 14

A proton is released with a velocity of 1000 meters per second (m/s) toward the sky in a strong magnetic field that points toward the east. What happens to the proton after it is released? (a) Nothing. (b) The proton decays into a neutron, a positron, and a neutrino. (c) The proton races off toward the north. (d) The proton keeps moving at 1000 m/s but veers north, eventually moving in a circle. (e) The proton gradually stops spinning.

Question 15

A proton is released with a velocity of 1000 m/s in a certain direction in empty space far, far away from any star, planet, or gas. What happens to the proton after it is released? (a) Nothing: the proton continues to move at the same speed and in the same direction until eventually it is scattered by encounters with photons, stars, planets, or gases. (b) The proton decays into a neutron, a positron, and a neutrino. (c) The proton races off toward the north. (d) The proton keeps moving at 1000 m/s but veers west, moving in a circle. (e) The proton gradually slows down because it runs out of momentum.

Question 16

In empty space far, far away from any star, planet, or gas, two protons are released at speed zero with a separation of 1 millimeter. What happens? (a) Nothing. (b) They fly apart in opposite directions along the line that connects them because their electric repulsion is much greater then their gravitational attraction. (c) They fly towards each other along the line that connects them. (d) They orbit each other in perfect circles, never colliding. (e) They collide and change into a deuteron and a positron and an anti-neutrino.

Question 17

In empty space far, far away from any star, planet, or gas, a proton and an electron are released at speed zero with a separation of 2 millimeters. What happens? (a) Nothing. (b) They fly apart in opposite directions along the line that connects them. (c) They fly towards each other along the line that connects them, and, after radiating some photons, they settle down to form a hydrogen atom. (d) They orbit each other in perfect circles, never colliding or radiating. (e) They collide and change into a neutron and an anti-neutrino.

Question 18

A molecule of a carbohydrate like sugar is a chemical rearrangement of some carbon atoms and some water molecules. Concentrated sulfuric acid strongly attracts water molecules and can extract them from some molecules like sugar that contain water. What happens if concentrated sulfuric acid is mixed with an equal amount of dry sugar? (a) nothing, (b) the sugar dissolves in the acid, (c) the acid sucks the water out of the sugar, leaving mostly carbon atoms, which rise in a hot, black column that gives off a sweet-smelling steam loaded with acid rain, (d) the sugar dries up the acid, (e) the beaker holding the solution combines with the acid and the sugar to form silicon.

RNA and Protein

To make a protein, RNA polymerase copies the DNA of the gene for the protein onto messenger RNA, called mRNA. Ribosomes then latch onto the mRNA and find a fiducial sequence, which tells them where to start translating the mRNA into a chain of amino acids. Each ribosome starts at the first “start codon,” AUG, after the fiducial sequence and proceeds in groups of three bases until it reaches one of the “stop” codons, which it does not translate. For instance, according to the genetic code (last page of final exam), the sequence AUGGGAUAG codes for AUG which is Met (short for methionine) and for GGA which is Gly (short for glycine) and then stops before coding the stop codon UAG. So in this example, the peptide is Met-Gly, which is too short to be considered to be a protein. Here and in what follows, I will write only the part of the mRNA that starts with the correct start codon, AUG. In some cases, cells remove the Met from the front end of the protein.

Question 19

What peptide does AUGCUAGCUCCUUAG code for? (a) Met-Gly-Asp-Phe, (b) Ser-Ile-Val-Asn, (c) Met-Cys-Thr-Trp, (d) Met-Arg-Pro-Glu-Thr, (e) Met-Leu-Ala-Pro.

Question 20

Suppose the DNA of the cell mutated to AUGACUAGCUCCUUAAUG, in which an extra A is inserted after the start codon and an extra AU at the end to make a stop codon? Note how disastrous the extra base, here A, is. (a) Met-Thr-Ser-Ser-Leu, (b) Met-Ala-Cys-Asn-Met, (c) Met-Cys-Trp-Val-Ser, (d) Met-Pro-Pro-Glu-Thr, (e) Met-Asp-Ala-Pro-Asn-Pro.